This is a digital copy of a book that was preserved for generations on library shelves before it was carefully scanned by Google as part of a project to make the world's books discoverable online.

It has survived long enough for the copyright to expire and the book to enter the public domain. A public domain book is one that was never subject to copyright or whose legal copyright term has expired. Whether a book is in the public domain may vary country to country. Public domain books are our gateways to the past, representing a wealth of history, culture and knowledge that's often difficult to discover.

Marks, notations and other marginalia present in the original volume will appear in this file - a reminder of this book's long journey from the publisher to a library and finally to you.

Usage guidelines

Google is proud to partner with libraries to digitize public domain materials and make them widely accessible. Public domain books belong to the public and we are merely their custodians. Nevertheless, this work is expensive, so in order to keep providing this resource, we have taken steps to prevent abuse by commercial parties, including placing technical restrictions on automated querying.

We also ask that you:

+ *Make non-commercial use of the files* We designed Google Book Search for use by individuals, and we request that you use these files for personal, non-commercial purposes.

+ *Refrain from automated querying* Do not send automated queries of any sort to Google's system: If you are conducting research on machine translation, optical character recognition or other areas where access to a large amount of text is helpful, please contact us. We encourage the use of public domain materials for these purposes and may be able to help.

+ *Maintain attribution* The Google "watermark" you see on each file is essential for informing people about this project and helping them find additional materials through Google Book Search. Please do not remove it.

+ *Keep it legal* Whatever your use, remember that you are responsible for ensuring that what you are doing is legal. Do not assume that just because we believe a book is in the public domain for users in the United States, that the work is also in the public domain for users in other countries. Whether a book is still in copyright varies from country to country, and we can't offer guidance on whether any specific use of any specific book is allowed. Please do not assume that a book's appearance in Google Book Search means it can be used in any manner anywhere in the world. Copyright infringement liability can be quite severe.

About Google Book Search

Google's mission is to organize the world's information and to make it universally accessible and useful. Google Book Search helps readers discover the world's books while helping authors and publishers reach new audiences. You can search through the full text of this book on the web at `http://books.google.com/`

Bt. from Sexton
First English Lot
2 Vols

600050980S

10/- 6V

Richard Brinsley Sheridan.
K. 2

C
1841 e. 22/1

THE

SYSTEM OF THE WORLD.

BY

P. S. LAPLACE,

MEMBER OF THE NATIONAL INSTITUTE OF FRANCE.

TRANSLATED FROM THE FRENCH

BY

J. POND, F.R.S.

IN TWO VOLUMES.

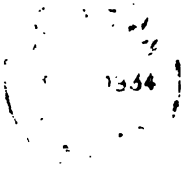
VOL. I.

LONDON:

PRINTED FOR RICHARD PHILLIPS, BRIDGE STREET, BLACKFRIARS.

1809.

W. Flint, Printer, Old Bailey.


1954

ADVERTISEMENT.

In this work I shall adopt the division of the quadrant into a hundred degrees, each degree being divided into a hundred minutes, and each minute into an hundred seconds. Temperature will be referred to the mercurial thermometer divided into one hundred degrees from the freezing to the boiling point of water, under a pressure equivalent to the weight of a column of mercury of the height of sixty-six centimetres. And I shall refer all linear measures to the metre, determined by the arc of the terrestrial meridian included between Dunkirk and Barcelona.

NOTE—BY THE TRANSLATOR.

THE reader will find the angular measures and measures of time used by the author reduced in the margin to the sexigesimal system adopted in this country; this was thought better than altering the text of an original work of such importance.

ERRATA.

VOL. I. P. 126, *for* sexigonal, *read* sexigesimal.
VOL. II. P. 93, line 2, *for* 120° *read* 2° and its reduction 1° 48′.

Table of French Measures.

	English inches
Millimetre - -	0.03937
Centimetre - -	9.39371
Decimetre - -	3.93710
Metre, 3.281 feet - -	39.37100
Decametre - -	393.71000
Hecatometre - -	3937.10000
Chiliometre - -	39371.00000
Myrcometre - -	393710.00000
The old French foot -	12.78933

TABLE OF CONTENTS,

VOLUME THE FIRST.

Advertisement iii
Note..... iv

BOOK I.

Of the apparent Motions of the Celestial Bodies 1

CHAP. I.

Of the diurnal motion of the Heavens..... 3

CHAP. II.

Of the Sun and its proper Motion..... 6

CHAP. III.

Of Time and its Measure..... 29

CHAP. IV.

Of the Moon's Motion, of its Phases, and of Eclipses..... 40

CHAP. V.

Of the Planets, and particularly of Mercury and Venus..... 66

CHAP. VI.

Of Mars..... 76

CHAP. VII.

Of Jupiter and its Satellites..... 79

CHAP. VIII.

Of Saturn, its Satellites and its ring..... 88

CHAP. IX.

Of Uranus and his Satellites..... 95

CHAP. X.

Of Comets..... 97

CHAP. XI.

Of fixed Stars and their Motions..... 99

CHAP. XII.

Of the Figure of the Earth, and of the Variation of Gravity at its Surface..... 113

CHAP. XIII.

Of the ebbing and flowing of the Sea..... 165

CHAP. XIV.

Of the terrestrial Atmosphere, and astronomical Refraction..... 177

BOOK II.

Of the real Motions of the celestial Bodies.... 205

CHAP. I.

Of the rotatory Motion of the Earth..... 207

CHAP. II.

Of the Motion of the Planets round the Sun.. 213

CHAP. III.

Of the Motion of the Earth round the Sun..... 218

CHAP. IV.

Of the Appearances owing to the Motion of the Earth..... 232

CHAP. V.

Of the Figures of the Planets, and of the Law of their Motion round the Sun..... 240

Table of the elliptic Motions of the Planets.... 252

CHAP. VI.

Of the Figure of the Orbits of Comets, and of the Laws of their Motion round the Earth... 257

CHAP. VII.

Of the Laws of the Motion of Satellites round their Planets..... 270

BOOK III.

Of the Laws of Motion..... 288

CHAP. I.

Of Forces and their composition..... 290

CHAP. II.

Of the Motion of a material Point..... 295

CHAP. III.

Of the Equilibrium of a System of Bodies..... 329

CHAP. IV.

Of the Equilibrium of Fluids..... 347

CHAP. V.

Of the Motion of a System of Bodies..... 357

THE

SYSTEM OF THE WORLD.

Me vero primum dulces ante omnia musæ
Quarum sacra fero, ingenti perculsus amore
Accipiant, cœlique vias, et sidera monstrent.

Of all the natural sciences, Astronomy is that which presents the longest series of discoveries. There is an immense distance from the first view of the heavens, to that general view by which, at the present day, we comprehend the past and future state of the system of the world. To arrive at this it was necessary to observe the heavenly bodies during a long succession of ages, to recognize from their appearances the real motion of the Earth, to develope the laws of the planetary motions, and

from these laws to derive the principles of universal gravitation, and to redescend from this principle to the complete investigation of all the celestial phenomena, even in their minutest details. This is what the human understanding has accomplished in astronomy. The exposition of these discoveries, and of the most simple manner in which they may arise one from the other, would have the double advantage of presenting a great assemblage of important truths, and the true method which should be followed in investigating the laws of nature. This is the object I propose in the following work.

BOOK THE FIRST.

OF THE APPARENT MOTIONS OF THE HEAVENLY BODIES.

CHAP. I.

Of the diurnal Motion of the Heavens.

If in a fine night, and in a place where the horizon is uninterrupted, we follow with attention the appearance of the heavens, it will be seen to vary at every instant. Some stars are rising above, others setting below the horizon; some begin to appear in the east, others disappear towards the west; several, as the pole star and the stars of the Great Bear, never reach the horizon. In these various motions, their respective positions to each other remain unchanged, and they describe circles, so much the less as they are nearer a point,

which seems to be immoveable. Thus the heavens appear to revolve round two fixed points, called from this circumstance, the POLES of the world, and in this motion is included the whole system of stars. The pole elevated above our horizon, is the north pole; the opposite pole, which we imagine beneath the horizon, is the south pole. Already several interesting questions present themselves to be resolved. What becomes during the day of the stars which we have seen in the night? From whence come those which begin to appear? Where are those gone, which have departed from our view? An attentive examination of these phenomena will afford a simple answer to these questions. In the morning, the brightness of the stars grows fainter as the dawning light increases, in the evening they become more brilliant as the twilight diminishes; it is not therefore because they cease to shine, but because they are effaced by the more vivid light of the sun, that we are unable to see them. The for-

tunate discovery of the telescope has enabled us to verify this explanation by shewing us the stars, even when the sun is at its greatest elevation above the horizon. Those which are near enough the pole never to reach the horizon appear constantly above it.

CHAP. II.

Of the Sun and of its proper Motion.

All the heavenly bodies participate in the diurnal motion of the celestial sphere; but many have proper motions of their own, which it is interesting to follow, because they alone can conduct us to the knowledge of the system of the world. In the same manner as in measuring the distance of an object, we observe it in two different positions, so, to discover the laws of nature, we must consider her under different points of view, and observe the development of those laws in the change of appearance which she presents to us. Upon the earth we vary these phenomena by experiments; in the heavens, we determine with care, all those which the celestial motions present to us. In thus interrogating nature, and submitting her answers to analysis, we can, by a train of reasoning and induc-

tion skilfully managed, arrive at the causes of these phenomena; that is to say, we can reduce them to general laws, from which the phenomena are derived. It is to discover these laws and to reduce them to the least possible number, that all our efforts should tend; for the first causes, and the intimate nature of beings, will be to us eternally unknown.

Of all the heavenly bodies which appear to have a motion of their own, the most remarkable is the Sun. Its proper motion in a contrary direction to the diurnal motion, manifests itself by the appearance of the heavens during the night, which changes and is renewed with the seasons. The stars situated in the path of the Sun and which set a little time after him, soon are lost in his light, and at length reappear before his rising; the Sun therefore advances towards them in a direction contrary to his diurnal motion; it is thus that for a long time his proper motion was examined, but

at present this motion is determined with great precision by observing every day the meridian altitude of the sun, and the interval of time which elapses between his passage and that of the stars over the meridian. We have thus the motion of the Sun in the direction of the meridian and likewise in the direction of the parallels; the resulting motion obtained by the combination of these two is its true motion. In this manner it has been found that the Sun moves in an orbit which at the commencement of 1750 was inclined to the equator * 26° 0796; which orbit has been named the Ecliptic. It is by the combination of the proper motion of the Sun with its diurnal motion that the changes of the seasons are produced. The points of intersection with the ecliptic and equator are called the equinoxes, for in fact the Sun in these two points describing the equator by its diurnal motion and this circle being divided into two equal parts by all the horizons, the day is then equal to the night in every

* 23° 28′ 18″.

part of the earth. In proportion as the Sun, in leaving the equinox of spring, advances in its orbit, its meridian altitudes above our horizon increase more and more ; the visible arc of the parallels it describes every day, continually augments, and increases the length of the days till the Sun has attained its greatest altitude. At this epoch the days are the longest in the year, and because near this *maximum* the variations of the meridian altitude of the Sun are insensible, the Sun (considering only that altitude on which the length of the days depend) appears stationary, for which reason this point of the *maximum* has been named the summer *solstice*. The parallels which the the Sun describes on that day is called the *tropic* of summer. It then redescends towards the equator which it traverses again at the autumnal equinox, and from thence it arrives at its minimum of altitude, or at the winter *solstice*. The parallel then described by the sun, is the winter *tropic*, and the day is the shortest of the year ;

arrived at this term the Sun again ascends and returns to the vernal equinox. Such is the constant progress of the Sun and of the seasons. The spring is the season comprised between the vernal equinox and the summer solstice; the interval from this solstice to the autumnal equinox forms the summer. Autumn is the interval from this equinox to the winter solstice; and the interval from this solstice to the vernal equinox, is winter.

The presence of the Sun above the horizon being the cause of heat, it might be imagined that the temperature would be the same in summer as in spring, and in the autumn as in the winter, but the temperature is not the instantaneous effect of the presence of the Sun, but the result of its long continued action. It does not in a day produce its maximum of effect till some time after his greatest altitude above the horizon, nor in a year till the solstitial altitude is passed.

The different altitudes of the pole in dif-

ferent climates produce in the seasons those remarkable varieties which we will now examine, from the equator to the poles. At the equator the poles are in the horizon, which there cuts all the parallels in two equal parts, the day is therefore always equal to the night, and at the equinoxes the Sun passes through the zenith of the place. The meridian altitudes of the Sun at the solstices are the least, and equal to the complement of the inclination of the ecliptic to the equator. The solar shadows in these two positions of the Sun are diametrically opposite, a circumstance which never occurs in our climates, where at noon they are always directed towards the north.

At the equator, therefore, properly speaking, there are two summers and two winters every year. The same thing takes place in every country, where the height of the pole is less than the obliquity of the ecliptic. Beyond this limit there is only one summer and one winter in every year, the Sun never reaches the zenith, the long-

est day in summer augments, and the shortest in winter diminishes, as we approach the pole; and when the zenith is only distant from it by a quantity equal to the obliquity of the ecliptic to the equator, the Sun never sets on the day of the Summer solstice, nor rises at the winter solstice; still nearer the poles, the time of its presence and of its absence on the horizon exceeds several days and even months. Finally under the pole the horizon being the equator itself, the Sun is always above the horizon when on the same side of the equator, and always below it when on the opposite side; so that there is but one day and one night throughout the year.

The intervals which separate the equinoxes and the solstices are not equal: that from the vernal equinox to the autumnal is about seven days longer, than from the autumnal to the vernal; the proper motion of the Sun, therefore, is not uniform;—accurate and multiplied observations have taught us, that it is the most rapid in a cer-

tain point in the solar orbit, situated near the winter solstice, and slowest in an opposite point situated near the summer solstice; The Sun describes in a day * 1°1327, in the first point, and only † 1°0591 in the second; thus during the course of a year the Sun's daily motion varies from the greatest to the least, by three hundred and thirty-six thousandths of its mean value.

To obtain the law of this variation, and in general that of all the periodical inequalities, the following consideration has been made use of, since the sines and cosines of angles become the same at every circumference to which they arrive, they are proper to represent these inequalities; in this manner, therefore, all the inequalities of the celestial motions have been expressed, and it only remains to separate these inequalities from each other, and to determine the angles on which they depend. In this manner it has been

---

* 1° 1′ 9″9. † 57′ 11″ 5.

found that the variation of the angular velocity of the Sun is very nearly proportional o the cosine of its mean angular distance from the point where its velocity is the greatest.

It is natural to think that the distance of the Sun from the Earth varies, with its angular velocity, and it has been proved to do so by the measures of its apparent diameter. This augments and diminishes in the same time and according to the same law as this velocity, but in a ratio only half as great; when the velocity is greatest the apparent diameter is * 6035″,7; when the velocity is least it is only † 5836″,3, thus its mean diameter is ‡ 5936″; this quantity should be diminished a few seconds, to allow for the effect of irradiation which dilates a little the apparent diameters of luminous bodies.

The distance of the Sun from the Earth, being reciprocally as its apparent diameter, its increase follows the same law as the

---

* 32′ 35″ 5 † 28′ 49″ ‡ 30. 42,25.

diminution of its diameter. We name *Perigee* the point of the orbit in which the Sun is nearest to the Earth, and *Apogee* the opposite point in which it is the most remote. It is in the first of these points that the Sun has both the greatest apparent diameter, and the greatest velocity; at the second point both the diameter and the velocity are at a minimum. To explain the diminution of the apparent distance of the Sun it is sufficient to suppose it farther from the Earth, but if the variation in its motion arose from this cause alone, and if the real velocity of the Earth in its orbit was constant, its apparent velocity should be diminished in the same proportion as its apparent diameter, but it diminishes in a ratio twice as great; there is therefore an actual retardation in the motion of the Sun, as it recedes from the Earth. By the combined effect of this retardation of the velocity and augmentation of the distance, the angular motion in a day diminishes as the square of the distance increases, so that its product by this square is very nearly constant.

All the measures of the apparent diameter of the Sun, compared with the observations of its daily motion, confirm this result.

Let us imagine a straight line joining the centres of the earth and Sun, and call it the RADIUS VECTOR of the Sun, it is easy to see that the small sector, or area traced in a day by this radius round the Earth, is proportional to the product of the square of this area by the daily motion of the Sun: thus this area is constant, and the whole area traced by the radius vector, reckoning from a fixed radius, increases as the number of days elapsed from the epoch on which the Sun was upon this radius. From hence results this remarkable law in the motion of the Sun; namely that, *The area described by its radius vector are proportional to the times.*

If, from these data, we every day set down the position and length of the radius vector of the solar orbit, and then make a curve pass through the extremities of these

radii, we shall perceive that this curve is not exactly circular, but that it is somewhat elongated in the direction of the straight line which, passing through the centre of the Earth, joins the points of the greatest and least distance of the Sun. The resemblance of this curve to an ellipse, having led to a comparison with it, their identity has been recognised, from whence this conclusion has been established, *that the solar orbit is an ellipse, of which the centre of the earth occupies one of the foci.* The ellipse is one of those celebrated curves, both in ancient and modern geometry, which being formed by the section of a surface of the cone by a plane, have been called *conic sections.* It is easy to describe it by fixing the extremities of a thread upon two immovable points, which thread being stretched upon a plane by a point which slides along it, the curve traced by the point in this motion is an ellipse, it is evidently elongated in the direction of the right line which joins the foci, and

which being prolonged on each side till it meets the curve forms the greater axis, whose length is equal to that of the thread. The greater axis divides the ellipse into two equal and similar parts, the lesser axis is the straight line drawn through the centre and prolonged each way till it meets the curve; the distance from the centre to one of the foci, is the *excentricity* of the ellipse; when the two foci become united in one point the ellipse becomes a circle; by separating them the ellipse gradually lengthens, and when the distance of the foci becomes infinite, the distance of the focus to the nearest summit of the curve remains finite, and the ellipse becomes a parabola.

The solar ellipse differs but little from a circle, for its excentricity is evidently the excess of the greatest above the mean distance of the Sun from the Earth; which excess we have seen is equal to one hundred and sixty-eight ten-thousandths of this distance. Observations seem to indicate in this excentricity, a very slow diminution, and scarcely perceptible in a century.

To have a just idea of the elliptic motion of the Sun, let us conceive a point moved uniformly in the circumference of a circle, of which the centre is that of the Earth, and whose radius is equal to the distance perigee of the Sun.

Suppose, moreover, that this point and the Sun, set off together from the perigee, and that the angular motion of the point is equal to the mean angular motion of the Sun. Whilst the radius vector of the point revolves uniformly round the Earth, the radius vector of the Sun moves in an unequal manner, by forming always with the distance perigee, and the arcs of the ellipse, sectors proportional to the times. At first it will precede the radius vector of the point, and make an angle with it, which after having augmented to a certain limit will diminish and become nothing; when the Sun is at the apogee, then the two radii will coincide with the greater axis. In the second half of the ellipse the radius vector of the point will precede the

radius vector of the Sun and form with it angles, which are exactly the same as in the first half, at the same distance from the perigee, where it will again coincide with the radius vector of the Sun and the greater axis of the ellipse. The angle by which the radius vector of the Sun precedes the radius vector of the point is called the *equation of the centre*; its maximum, or greatest equation of the centre, was in 1750 equal * 2°1409. The angular motion of the point round the earth is concluded from the length of an entire revolution of the Sun in its orbit; by applying to this the equation of the centre, we obtain the angular motion of the Sun. The investigation of this equation is an interesting problem in analysis, which cannot be solved but by approximation; but the small excentricity of the solar orbit, leads to series, which converge rapidly, and are easily reduced to the form of tables.

---

* 1 55′ 36″ 4.

The position of the greater axis of the solar ellipse is not constant. The angular distance of the perigee, to the equinox of spring reckoned in the direction of the Sun's motion was in the beginning of 1750 = *309° 5790, but it has relatively to the fixed stars an annual motion of about †, 36″7, in the same direction as that of the Sun.

The solar orbit approaches by insensible degrees to the equator; the secular diminution of its obliquity to the plane of this great circle may be estimated at about ‡ 154″ 3.

The elliptic motion of the Sun, will not exactly represent modern observations; their extreme precision has discovered small inequalities of which it would have been almost impossible by observation alone to have developed the laws. The investigation of these inequalities belongs to that branch of astronomy, which re-descends from

---

* 278° 37′ 16″. † 11″ 9. ‡ 50″.

causes to phenomena, and which will form the object of the fourth book.

The distance of the Sun to the Earth has interested astronomers at every period. They have endeavoured to measure it by all the means which astronomy has successively indicated. The most natural and the most simple is that which geometricians apply to measure the distance of terrestrial objects. From the two extremities of a known base they observe the angles which it makes with the visual rays of the object, deducting their sum from two right angles; they obtain the angles which these rays make at their point of union; this angle is what is called the parallax of the object, whose distance from the extremities of the base is by this means easily deduced.

In applying this method to the Sun, it is desirable to choose a base of the greatest extent that can be taken on the surface of the Earth. Let us imagine two observers placed under the same meridian and observ-

ing at the same instant the meridian altitude of the centre of the Sun and its distance from the same pole; the difference of these two observed distances, will be the angle, which the straight line joining the observers would subtend if seen from the centre of the Sun. The position of the observers gives this straight line, in parts of the terrestrial radius; it will therefore be easy to conclude from these observations the angle which the semi-diameter of the earth would subtend as seen from the centre of the Sun. This angle is called the parallax of the Sun, but it is too small to be determined by this method, which can only enable us to judge that its distance is at least six thousand diameters of the earth. We shall see hereafter, that astronomical discoveries furnish other methods much more exact of determining the parallax of the Sun, which we now know to be very nearly *27″,2, at the mean distance of the

* 8″ 8.

Sun from the Earth, from whence it follows that this distance is 23405 times the radius of the earth. The smallness of the parallax of the Sun proves its immense magnitude; we are certain that at the same distance at which this luminary is seen under an angle of * 5936″, the earth would not appear under an angle of † 100 seconds, and the volumes of spherical bodies being as the cubes of their diameters, the volume of the Sun is at least two hundred thousand times greater than that of the Earth; it is about thirteen hundred thousand times bigger if, as the observations seem to indicate, the parallax is only ‡ 27″2.

Black spots are observed at the surface of the Sun of an irregular form, their number, position and magnitude are very variable—they are often very numerous and of considerable extent; some have been observed whose diameter exceeded four or five times that of the Earth; sometimes,

---

* 32′ 3″ 2. † 32″ 4. ‡ 9″.

though rarely, the Sun has appeared pure and without spots for several years together. The solar spots are almost always surrounded by a penumbra, which is inclosed in a cloud of light, more brilliant than the rest of the Sun, and in the midst of which the spots are seen to form and disappear. All this indicates that at the surface of this enormous mass of fire, vivid effervescences take place, of which our volcanoes form but a feeble representation. But whatever be the nature of the solar spots, they have made us acquainted with a remarkable phenomenon—the rotation of the Sun.

Amidst all their variations we can discover regular motions, which are exactly the same as the corresponding points of the surface of the Sun, if we suppose it to have a motion of rotation on an axis almost perpendicular to the ecliptic in the direction of its motion round the earth. From the continued observation of these spots the rotation of the Sun has been estimated

at about twenty-five days and a half; that the solar axis is inclined * $8°\frac{1}{7}$, to the plane of the ecliptic, and that the points of this equator when they ascend by their motion of rotation above this plane (i.e. the ecliptic) towards the north pole, intersect it in a point which, seen from the centre of the Sun, was † 86°20 distant from the vernal equinox in 1750.

The spots on the Sun are almost always comprised in a zone of its surface, whose breadth measured on the solar meridian extends only ‡ 33° or 34° on each side of the equator;—they have sometimes however been seen as far as § 44°. Bouguer found, by some curious experiments on the intensity of light on different parts of the Sun's disk, that this light was more intense at the centre than near the limb. Yet the same portion of the disk being carried round

---

* 7″ 28′ 12″. † 77° 34′ 48″. ‡ 29° 42′ or 30° 36′. § 39° 36′.

from the centre towards the limb, presents itself to us under a smaller angle; its light therefore should be more intense: it must therefore in some measure be extinguished, which can only be explained by supposing it surrounded by a dense atmosphere, which, being obliquely traversed by the rays emanating from the limb, weakens them more than those which proceeding from the centre pass it perpendicularly. Thus a solar atmosphere is indicated by this phenomenon with considerable probability.

It is the general opinion that it is this atmosphere which reflects to us that faint light which is visible, particularly about the vernal equinox, a little before the rising or after the setting of the Sun, and to which has been given the name of zodiacal light, (*lumiere zodiacale.*) The fluid which transmits it to us is extremely rare, since the stars are visible through it; its colour is white, and its apparent figure that of a cone whose base is applied to the Sun; such

would be the appearance of an ellipsoid of revolution extremely flattened, whose centre and plane of its equator should be the same with those of the Sun. The length of the zodiacal light sometimes subtends an angle of more than * 100°; we shall see in the course of this work that the atmosphere of the Sun does not extend to so great a distance, it is not therefore this atmosphere that reflects the zodiacal light. Dominique Cassini, who was the first person who described this light correctly, remarked that it was weakened when the Sun had few spots, from which he conjectured that both the spots and the light arose from the same source, by the expansive force of the Sun which throws to its surface the dense matter of the spots, and which projects to a distance the rare and transparent matter of the zodiacal light. But the true cause of this light is still unknown.

---

* 90°.

CHAP. III.

Of Time and its Measure.

TIME relatively to us, is the impression which a series of objects leaves upon the memory, and of which we are certain the existence has been successive. Motion is a proper measure of time; for since a body cannot be in several places at the same time, it can only move from one place to another by passing through all the intermediate points. If we are assured that at every point of the line it describes, it is animated by the same force, it will describe it with an uniform motion, and the portions of this line will measure the time employed to describe them. When a pendulum at the end of every oscillation is in exactly the same position, the lengths of its oscillations are the same, and time may be measured by their number. We may employ

likewise for this purpose the successive revolutions of the celestial sphere which appear perfectly uniform. But mankind have universally agreed for this purpose to make use of the motion of the Sun.

In civil life, the day is the interval of time which elapses between the rising and setting of the Sun; the night is the time during which the Sun remains below the horizon. The *astronomical day* comprehends the period of its diurnal revolution; it is the interval of time between two consecutive noons or midnights. It surpasses the length of a revolution of the heavens, which forms a *siderial day;* for if the Sun passes the meridian at the same instant as a star, the day after it will pass later in consequence of its proper motion by which it advances from west to east, and in the space of a year it will pass the meridian once less than the star; it is found that taking for unity the mean astronomical day, the sidereal day is *0.997269722. The

---

*23h 56′ 4″ 1.

astronomical days are not equal: two causes combine to produce their difference, the inequality of the proper motion of the Sun, and of the obliquity of the ecliptic. The effect of the first cause is sensible: thus at the summer solstice, near which the motion of the Sun is the slowest, the astronomical day approaches more to the siderial day than the winter solstice, where this motion is more rapid.

To conceive the effect of the second cause it must be noticed, that the excess of the astronomical day above the siderial day is only due to the proper motion of the Sun reduced to the equator. If we imagine two great circles of the celestial sphere to pass through the extremities of the small arc which the Sun describes in the ecliptic in one day, and likewise through the poles of the world, the arc of the equator which they intercept is the daily motion of the Sun reduced to the equator; and the time this arc employs to traverse the meridian is the excess of the astronomical above

the siderial day; now it is evident that at the equinoxes the arc of the equator is less than the corresponding arc of the ecliptic, in the ratio of the cosine of the obliquity of the ecliptic to radius: at the solstices it is greater in the ratio of radius to the cosine of the same obliquity; the astronomical day, therefore, is diminished in the first case and augmented in the second. To obtain a mean day, independent of these causes, we imagine a second sun moved uniformly on the ecliptic, and always passing the greater axis of the solar orbit at the same instant as the true sun, which will cause the inequality arising from the proper motion of the Sun to disappear. The effect arising from the obliquity of the ecliptic is then made to disappear by imagining a third sun to pass the equinox at the same instant as the second sun, and to move along the equator in such a manner that the angular distances of these two suns from the vernal equinox may always be equal to each other. The interval of time between two consecutive transits of this

third sun over the meridian forms the *mean astronomical day*. *Mean time* is measured by the number of these transits; *true time* or *solar time* by the number of transits of the real sun over the meridian. The arc of the equator intercepted by two meridians drawn through the centres of the real sun, and the third sun reduced into time, in the proportion of the whole circumference to one day, is that which is called the *equation of time*.

The returns of the Sun to the same equinox mark the years in the same manner as its returns to the meridian mark the days. In consequence of its annual motion it employs *365.242222 to return to the vernal equinox. This period forms the *tropical year*. Observation shews us that he employs a longer time to return to the same stars. The siderial year is the interval between two of these consecutive returns; it is greater than a tropical year

---

* $365^d\ 5^h\ 48'\ 4''$.

by * 0, 01462 of a day. Thus the equinoxes have a retrograde motion on the ecliptic, or in a direction contrary to that of the Sun, by which they describe every year an arc equal to the mean motion of the Sun in the interval of 0,01462 or 155″09.† For the uses of society different periods have been imagined to measure the portions of this revolution. Nature offers two very remarkable ones in the returns of the Sun to the meridian, and to the same equinox; but both require to be divided into smaller periods. The division of the day into ten hours, the hour into a hundred minutes, and the minute into a hundred seconds, is the most simple; it is natural to make the astronomical day commence at midnight, that it may comprehend in its duration all the time of the presence of the Sun above the horizon. It is convenient to fix the origin of the year at the vernal equinox, the period in which nature begins

---

* 21′ 3″ 1. † 50″ 2.

to revive; the seasons divide the year into four parts, which are each again subdivided into three months of thirty days. Each month has been divided into three periods of ten days or *decades*. In this manner the civil year would be divided into 360 days, but we have seen that it really consists of above 365; these days of excess are called complementary. Notwithstanding in this system of dividing the year, the order of things relative to the days of the decade will be something deranged by the complementary days; the correspondence of the days of the decade with the days of the month, and of the decadary festivals with that of the seasons, will render it preferable to the smaller division into weeks which are independent of the months.

If the length of the year was fixed at 365 days, its commencement would continually anticipate that of the tropical year, and the months would pass through all the different seasons with a retrograde motion in a period of about 1320 years. This

method, formerly in use in Egypt, deprives the calendar of the advantage of fixing the months and festivals at the seasons, and of rendering them useful epochs for the purposes of agriculture. This great advantage may be preserved to the inhabitants of the country, by considering the origin of the year as an astronomical phenomenon, which should be fixed by observation and calculation at the moment of midnight, which precedes the vernal equinox; but then the years would lose the advantage of being regular periods of time, easy to subdivide into days, and introduce some confusion into history and chronology, and sometimes render even the origin of the year uncertain, which we always require should be known in advance. To obviate these conveniences, and to fix the same months and festivals at the same seasons, intercalations have been contrived. The most simple of all is that which the Persians adopted in the eleventh century; it consists in adding seven times following six complementary

days instead of five every four years; and only to make this addition the eighth time every fifth year. The years thus augmented were called sextile, to distinguish them from the others, called common. According to this mode of intercalation, in thirty-three years, eight are sextile and twenty-five are common. This supposes the length of the 365 $\frac{8}{33}$ days greater by * 0.000202 of a day than the tropical year determined by observations; but a great number of centuries must elapse before the origin of the year would be so far removed from the equinox as to be sensible to the agriculturist.

It is much to be wished that all nations would adopt one common æra, not depending on moral revolutions, but determined by astronomical phenomena alone. We might fix its origin in the year in which the apogee of the solar orbit coincided with the summer solstice, which happened

---

* 1774.

about the year 1250; we should take for this origin the moment of the mean vernal equinox, which in that year answered to the 15th March,* 5^h 3676 at Paris. The universal meridian, from which terrestrial longitudes should be reckoned, should be that of the place at which it was midnight at that instant, and which is to the east of Paris † 185°2960. If after a long succession of ages, the origin of the æra should become uncertain, it would be difficult to recover it exactly by the motion of the apogee, considering the slowness and the irregularity of this motion; but we should have no uncertainty as to this origin, or as to the position of the universal meridian, upon calling to mind that at the moment of the mean equinox the mean longitude of the moon was ‡ 143°7714. Thus whatever is arbitrary in the origin of time, and of terrestrial longitudes might be made

* 12^h 52′ 56″ 0. † 166° 51′ 59″ 0.
‡ 129° 23′ 39′ 3.

to disappear. By afterwards adopting the preceding intercalation and division of the year, month, and day, we should obtain the most natural and simple calendar that can be suited to the inhabitants of this side of the equator.

One hundred years form a century, which is the longest period hitherto employed in the division of time.

CHAP. IV.

Of the Motion of the Moon, its Phases and Eclipses.

After the Sun, the Moon of all the heavenly bodies is that which interests us the most; its phases afford us a measure of time so remarkable that it has been primitively in use among all people. The Moon, like the Sun, has a proper motion from west to east, the length of its siderial revolution was * 27°32166118036, at the commencement of 1700; it is not the same for every century. The comparison of modern with ancient observations shews incontestably an acceleration in the mean motion of the Moon. This acceleration, though but little sensible since the most ancient recorded eclipse, will be developed in progress of time. But will it for ever continue

* $27^d\ 7^h\ 43'\ 11''\ 5$.

to increase, or will it stop and be changed into a retardation? this cannot be determined by observations, except after an immense number of ages; fortunately the discovery of its cause has anticipated them and shewn that it is periodical. The Moon moves in an elliptic orbit, of which the Earth occupies one of the foci. Its radius vector describes about this point equal areas in equal times. The mean distance of the Moon from the Earth being taken as unity, the excentricity of its ellipse is 0.0550368, which gives for the greatest equation of its centre *7°0099. The lunar perigee has a direct motion, that is, in the same direction as the motion of the Sun, and the length of its sidereal revolution is † 3232,46643 days.

These laws, analogous to those of the solar motion, are very far from being sufficient to represent the observations; the lunar motion is subject to a great many other irregularities, which are evidently con-

---

* 6° 18′ 32″. † 9ʸ 312ᵈ 11ʰ 11′ 29″ 5.

nected with the position of the Sun. We shall indicate the three principal. The most considerable of all, and that which was first recognized, is what we now call the evection; this inequality, which amounts to * 1°4902 is proportional to the sine of double the mean angular distance of the Moon from the Sun, *minus* the mean angular distance of the Moon from the perigee of its orbit. In the oppositions and conjunctions of the Moon with the Sun, it is confounded with the equation of the center, which it constantly diminishes, and for this reason the ancient astronomers who only determined the elements of the lunar theory by the means of eclipses, and with a view of predicting these phenomena, always found the equation of the centre less than the truth by the whole quantity of the evection. We observe likewise in the lunar motions a great inequality, which disappears in the conjunctions and oppositions, and also in those points where

* 1° 20′ 28 2.

the Sun and Moon are distant from each other * 100°. It is at its maximum and amounts to † 0°6608 when their mutual distance is ‡50°; from whence it is inferred that it is proportional to the sine of double the mean distance of the Moon from the Sun: this inequality, which is called the *variation*, disappears in eclipses, and therefore could not have been discovered by the observation of those phenomena.

Lastly, the motion of the Moon is accelerated when that of the Sun is retarded, and reciprocally from which another inequality arises known by the name of the annual equation, and of which the law is exactly the same with that of the equation of the centre of the Sun, but with a contrary sign. This inequality, which at its maximum is § 0°2064, in eclipses becomes confounded with the equation of the centre of the Sun, and in calculating the instant of these phenomena it is indifferent whether we consider separately these two equations, or sup-

---

* 90°. † 35′ 41″. ‡ 45°. § 11′ 8″ 7.

press the annual equation of the lunar theory, to augment the equation of the centre of the Sun. This is one of the principal reasons why the ancient astronomers gave too great a value to this last equation, at the same time that they assigned one too small to the equation of the center of the Moon, affected by the evection. The lunar orbit is inclined to the plane of the ecliptic* 5°7188; its points of intersection with it, called the *nodes*, are not fixed in the heavens, they have motion contrary to that of the Sun; which it is easy to recognize by the succession of stars the Moon meets with in traversing the ecliptic. The length of the sidereal revolution of these nodes is † 6793^{d} 3009. The ascending node is that in which the Moon rises above the ecliptic to advance towards the north pole; the *descending node* is that in which the Moon is descending towards the south. Their motion is subject to several inequalities, of which the greatest is proportional to the sine of

* 5° 8′ 49″. † 18^{y} 223^{d} 7^{h} 13′ 17″ 7.

double the angular distance of the Sun from the ascending node of the lunar orbit, and amounts to * 1° 8105 at its maximum. The inclination of the orbit is equally variable, its greatest inequality amounting to† 0°1631, is proportional to the cosine of the same angle, on which the inequality of the motion of the nodes depends.

The lunar orbit, and the orbits of the other heavenly bodies, have no more a real existence than the parabolas described by the projectiles at the Earth's surface. To represent the motion of a body in space we imagine a line passing through all the successive positions of its centre; this line is its orbit, whose plane is that which passes through two consecutive positions of the body, and at the same time through the centre, round which the motion takes place. Instead of conceiving the motion of a body in this manner, we may in imagination suppose it projected on a fixed plane and de-

*1° 37′ 46″ [redacted] 48″ 4.

termine the curve of projection and the height of the body above this plane. These different methods have their peculiar advantages, which, according to circumstances, may render one preferable to the other.

The apparent diameter varies in a manner analogous to the lunar motions, it is * 5438″, at its greatest distance, and † 6207 at its least.

The same means which were insufficient to determine the parallax of the Sun, on account of its smallness, have given that of the Moon equal to ‡ 10676″ at its distance from the Earth, which is an arithmetical mean between the two extremes; thus at the same distance at which the Moon appears to us to subtend an angle of § 5823″ the earth would be seen under an angle of ‖ 21352″, their diameters then are in the ratio of these numbers, or very nearly as 3 to 11, and the volume of the lunar globe

---

* 29′ 22″ 2. † 33′ 31″ ‡ 57′ 39″ § 31 26″ 6.
‖ 55′ 18″

is 49 times less than the volume of the Earth.

The phases of the Moon are among the most striking phenomena of the heavens. In disengaging itself in the evening from the rays of the Sun it re-appears with a slender crescent, which increases with its distance, and becomes an entire circle of light, when it is in opposition with the Sun. When it afterwards approaches to it, this circle is changed into a crescent which diminishes according to the same degrees by which it had increased, till in the morning it becomes immersed in the solar rays. The lunar crescent, always turned towards the Sun, evidently indicates that it is from the Sun it receives its light; and the law of the variation of its phases, which increase nearly as the versed sine of the angular distance of the Moon from the Sun, proves to us that it is spherical.

These phases are renewed at every conjunction, their return depends on the excess of the synodical motion of the Moon above

that of the Sun, which excess is called the synodical motion of the Moon. The length of the synodic revolution of the Moon, or the period of its mean conjunction is *29^{d}. 530588, it is to the tropical year nearly in the ratio of 19 to 235, that is 19 solar years form about 235 lunar months.

The *sysygies* are the points of the Moon's orbit, in which the Moon is in conjunction or opposition with the Sun; in the first point, the Moon is new, in the second full. The quadratures are the points of the orbit in which it is distant from the Sun † 100 or 300, reckoning in the direction of its proper motion. In those points which are called the first and second quarters of the Moon we see nearly half of its enlightened hemisphere, strictly speaking we see a little more, for when the exact half is presented to us the angular distance of the Moon from the Sun is a little less than ‡ 100 degrees. At this instant, which we recog-

---

* 9^{d} 12^{h} 44′ 2″. † 90° or 270°. ‡ 90°.

nise, because the enlightened is separated from the obscure part of the Moon by a straight line, the radius drawn from the observer to the centre of the Moon is perpendicular to that which joins the centres of the Moon and Sun.

Thus in the triangle formed by the straight lines which join these centres and the eye of the observer, the angle at the Moon is a right one; the distance therefore of the Earth to the Sun may be determined in parts of that of the Moon from the Earth. The difficulty of fixing with precision the instant in which we see the half of the lunar disk enlightened, renders this method not very exact; we owe to it nevertheless the first just notions that were formed of the immense magnitude of the Sun and of its distance from the Earth.

The explanation of the phases of the Moon leads us to that of eclipses, objects of terror to men in the times of ignorance, but of curiosity to philosophers in all ages. The Moon can only be eclipsed by the in-

terposition of some opake body which deprives it of the light of the Sun, it is evident that this opake body is the Earth, since an eclipse of the Moon never happens but at the oppositions, or when the Earth is between it and the Sun. The terrestrial globe projects behind it relatively to the Sun, a conical shadow, the axis of which is the straight line which joins the centres of the Sun and the Earth, and terminates in a point where the diameters of these two bodies are the same. These diameters seen from the centre of the Moon in opposition and at its mean distance are nearly *5920″ for the Sun and †21352″ for the Earth: thus the cone of the terrestrial shadow is at least three times longer than the distance of the Moon from the Earth; and its breadth at the points where it is traversed by the Moon is more than double the lunar diameter. The Moon, therefore, would be eclips-

---

* 31′ 58″. † 1° 55′ 18″.

ed every time she is in opposition with the Sun, if the plane of her orbit coincided with the ecliptic; but in consequence of the mu tual inclination of these planes, the Moon in the oppositions is often elevated above, or depressed below the lunar shadow, and does not enter it but when she is near the nodes. If the whole of the disk is immersed in the shadow, the eclipse of the Moon is said to be total; it is partial if only a portion of the disk is obscured; and we may conceive that the greater or less proximity of the Moon to the nodes at the moment of opposition may produce all the varieties which are observed in these eclipses.

The mean period of the revolution of the Sun is, with respect to the node of the Moon's orbit, * 346.61963 days; it is to the synodical revolution of the Moon nearly in the ratio of 223 to 19: thus after a period of 223 lunar months, the Sun and Moon return again to nearly the same posi-

* $346^{d}\ 14^{h}\ 52'\ 16''$.

tion relatively tothe nodes of the lunar orbit. As the eclipse should return again in the same order, this circumstance affords an easy manner of predicting them; but the inequalities of the motions of the Sun and the Moon produce very sensible differences, and besides the return of these two bodies to the same position with respect to the node in 223 months is not rigorously exact, the deviations which result, change at length the order of the eclipses that have been observed during one of these periods.

It is only in the conjunctions of the Sun and Moon that we can observe a solar eclipse, when the Moon interposed between the Earth and the Sun intercepts its rays. Although the Moon is incomparably smaller than the Sun yet by a remarkable circumstance, on account of its vicinity, its diameter differs but little from that of the Sun, it even happens from the variations in these diameters that they surpass alternately one another. Let us imagine the Sun and Moon in the same straight line with

the eye of the observer, he will see the Sun eclipsed, and if the apparent diameter of the Moon surpasses that of the Sun, the eclipse will be total, but if the diameter be less, he will see a luminous ring formed by that part of the Sun which extends beyond the disk of the Moon and the eclipse will be *annular;* if the Moon is not on the straight line which joins the centre of the Sun, and the observer, the Moon may then only conceal a part of the solar disk, and the eclipse will be partial Thus the difference in the distances of the Sun and Moon from the centre of the Earth, and of the proximity of the Moon to the node, must produce a great variety in the circumstances of a solar eclipse; to this may be added the elevation of the Moon above the horizon, which changes the angle of its apparent diameter, and which by the effect of the lunar parallax, may so augment or diminish the apparent distances of the Sun and Moon, that of two observers, one may see an eclipse of the Sun, which will

be totally invisible to the other. In this respect eclipses of the Sun differ from those of the Moon, which are the same to all places on the Earth.

We often see the shadow of a cloud transported by the winds rapidly pass over the hills and valleys, depriving those spectators which it reaches of the light of the Sun which others are enjoying; this is the exact image of a total eclipse of the Sun; a profound night which under favourable circumstances may last about five minutes accompanies these eclipses; the sudden disapparition of the Sun, with the sudden darkness that succeeds, fills all animals with dread; the stars which had been effaced by the light of day, show themselves in their full lustre, and the heaven resembles the most profound night: round the lunar disk, a crown of pale light has been perceived, which is probably the solar atmosphere, for its extent cannot accord with that of the Moon, as we are assured by eclipses of the Sun and

stars, that the lunar atmosphere is nearly insensible. The atmosphere which we may suppose to surround the Moon, inflects the luminous rays towards its centre, and if (as should be the case) the atmospherical strata are rarer in proportion as they are removed from the surface, these rays in penetrating into it, will be inflected more and more, and will describe a curve concave towards its centre. An observer in the Moon will not cease to see a star till it is depressed below the horizon an angle, called the *horizontal refraction.* The rays emanating from this star, seen at the horizon after having just touched the surface of the Moon, will continue their route by describing a curve similar to that by which they arrived; thus an observer placed behind the moon, relatively to the star, will see it in consequence of the inflection of the lunar atmosphere. The diameter of the Moon is not sensibly augmented by the refraction of its atmosphere; therefore a star eclipsed

by the Moon would be so, later than if this atmosphere did not exist; and for the same reason would sooner cease to be eclipsed: thus the effect of a lunar atmosphere would be principally perceived in the length of the eclipses of the Sun and stars by the Moon. Very exact and numerous observations have scarcely indicated a suspicion of this influence, and we may be assured that at the surface of the Moon, the horizontal refraction does not exceed * 5″. We shall see that at the surface of the Earth, this refraction is at least one thousand times greater; the lunar atmosphere, therefore, if any exist, must be of an extreme rarity, and even superior to that which we produce in our best-constructed air-pumps. From hence we may conclude that no terrestrial animal could live or respire at the moon, and that if it is inhabited, it must be by animals of another nature, and fluids being so

---

little compressed by an atmosphere so rare, would soon be reduced to vapours; there is then reason to think that all is solid at the surface of the moon, and this seems to be confirmed by the appearances which are seen in a good telescope, which shows it to us as an arid mass, on which some have thought they perceived the effects and even the explosion of volcanoes.

Bouguer has found by experiment that the light of the full Moon is about three hundred times fainter than that of the Sun. This is the reason why its light, collected in the focus of our largest mirrors, produces no sensible effect on the thermometer.

The Moon does not disappear entirely in its eclipses, but is still enlightened by a very faint light which comes to it by the Sun's rays inflected through the terrestrial atmosphere, and but for the great absorption of these rays by our atmosphere its brightness would be more vivid than when

at the full moon. It is evident that this light should be more considerable in the eclipse's apogee than perigee; the vapours and clouds may weaken it so much as to render the Moon in these eclipses sometimes invisible; and the history of astronomy affords us several examples, though rare, of the total disapparition of the Moon.

We may sometimes distinguish, particularly about the time of new Moon, that part of the lunar disk which is not enlightened by the Sun; this feeble light has been called *lumière cendrée*, and is the effect of the light which the illuminated hemisphere of the Earth reflects upon the Moon; what proves it is, that it is most sensible at the new moon, when the greatest part of this hemisphere is directed to the Moon; for it is clear that to a spectator in the Moon, the Earth will present a succession of phases, similar to that which the Moon presents to us, but accompanied by a much more intense light from the greater extent of the terrestrial surface.

The lunar disk contains a great number of invariable spots which have been observed and accurately described. They shew us, that this body always presents to us very nearly the same hemisphere; it turns then upon itself in a period equal to its rotation round the Earth, for if we imagine an observer placed at the centre of the Moon, supposed transparent, he will see the Earth and its visual ray revolve about him, and since this ray always intersects nearly the same point of the lunar surface, it is evident that this point must revolve round the spectator in the same time, and in the same direction as the Earth.

Nevertheless, continued observation of the lunar disk has discovered some small diversity in these appearances, the spots are perceived to approach to and recede alternately from the limb; those that are very near the extremity, appear and disappear successively by periodical oscilla-

tions, which have been distinguished by the name of the *libration of the moon.*

To form a just idea of the principal causes of this phenomenon, we should consider that the disk of the Moon, seen from the centre of the Earth, is terminated by a great circle of the lunar globe, perpendicular to the radius drawn from this centre to that of the globe. It is upon the plane of this great circle, that the hemisphere of the Moon is projected directed towards the Earth, and its appearances arise from its motion of rotation, relatively to its radius vector: if it was without a motion of rotation, this radius vector would trace, at every lunar revolution, the circumference of a great circle upon its surface, every point of which would be successively turned to us; but at the same time that the radius vector traces this circumference, the lunar globe by its revolution brings always very nearly the same point of its surface to this ra-

dius, and consequently the same hemisphere towards the Earth. The inequalities of the motion of the Moon produce some small variations in these appearances, for the motion of rotation not partaking in a sensible manner of these inequalities, it is variable with respect to its radius vector, which thus intersects its surface at different points. The lunar globe therefore makes, with respect to this radius, oscillations corresponding to the inequalities of its motion, which causes some part of its surface to be alternately concealed and exposed to our observation.

Moreover, the axis of rotation of the Moon is not exactly perpendicular to the plane of its orbit. In supposing it nearly fixed during a whole revolution of the lunar globe, it inclines more or less upon the radius vector of the Moon, and the angle formed by these two lines is acute during one half of the revolution, and obtuse during the other half; a spectator at the Earth sees, therefore, alternately one

and the other pole of rotation and the parts of the surface which are near it.

And lastly, the observer is not at the centre of the Earth, but at the surface; it is the visual ray drawn from his eye to the centre of the Moon, which determines the middle of the visible hemisphere, and it is clear that from the effect of the lunar parallax, this radius cuts the surface of the Moon at different points, according to the height of the Moon above the horizon.

All these causes produce only an apparent libration of the lunar globe, they are mere optical illusions, and do not affect its real motion of rotation; it is nevertheless true that this rotation may be subject to some small irregularities, but they have not yet been detected by observation.

It is not the same with the inequalities of the lunar equator. In endeavouring to determine its position by observations on the spots of the Moon, Dominique Cassini

was led to this remarkable result, which contains all the astronomical theory of the real libration of the Moon. If we conceive a plane to pass through the centre of the Moon, perpendicular to its axis of rotation, a plane which will be coincident with that of its equator, if moreover, we imagine a second plane, parallel to that of the ecliptic, and a third plane, which is the mean plane of the lunar orbit, these three planes will always have a common intersection. The second plane, situated between the two others, forms with the first an angle of about * 1°,67; by and with the second, an angle of † 5°.7188. Thus the intersections of the lunar orbit, with the ecliptic, or its nodes, always coincide with the mean nodes of the lunar orbit, and like them, have a retrograde motion, whose period is ‡ 6793^{d}.3009. In this

---

* 1° 30′ 10″ 8. † 5° 8′ 49″.

‡ 18^{y} 223^{d} 7^{h} 13′ 17″ 7.

interval, the two poles of the equator and of the lunar orbit, describe small circles parallel to the ecliptic, comprehending between them the poles of the ecliptic in such a manner, that these three poles are constantly situated on a great circle of the heavenly sphere.

Mountains of a very great height, rise up from the surface of the Moon; their shadows, projected on the plains, form spots which vary with the position of the Sun; upon the edge of the enlightened disk we see these mountains forming an indented border, extending beyond the line of light by a quantity which, being measured, indicates that their height is at least ten or twelve thousand feet. We recognize likewise by the direction of the shadows, that the surface is broken by cavities, nearly resembling the basons of our seas. Lastly, the lunar surface seems to shew the traces of volcanos; several observers have seen upon the unenlight-

ened part, a vivid light, which they have attributed to a volcanic eruption. We may likewise attribute to this cause, the formation of several new lunar spots.

CHAP. V.

Of the Planets, and in particular of Mercury and Venus.

In the midst of the infinite number of shining bodies, which are spread over the surface of the heavenly firmament, and which preserve among themselves a position nearly constant, we may perceive six stars moving in regulated periods and according to complicated laws, the investigation of which is one of the principal objects of astronomy. These stars, to which have been given the name of PLANETS, are Mercury, Venus, Mars, Jupiter, Saturn and Uranus; the two first never recede from the Sun beyond certain limits, the others are occasionally separated from him by all the angular distances possible. The

motions of all these bodies are comprehended in a zone of the celestial sphere called the zodiac, the breadth of which is about * 20°, and divided into two equal parts by the ecliptic.

The distance of Mercury from the Sun never exceeds † 32°; when it begins to appear in the evening it is with difficulty distinguished in the rays of twilight; it disengages itself more and more in the following days, and after arriving at about ‡ 25° from the Sun, it returns towards him again. In this interval the motion of Mercury with respect to the fixed stars is direct, but when in returning it comes within the distance of § 20° of the Sun, it seems stationary, after which its motion appears retrograde, it continues to approach the Sun, and is again in the evening lost in his rays. After continuing some time invisible, it is seen again in the morning, disen-

---

* 18°. † 28° 48′. ‡ 22° 30′. § 18°.

gaging itself from the Sun's rays and departing from the Sun, its motion is still retrograde as before its disapparition; arrived at the distance of 20° it is again stationary, then resumes its direct motion, its distance increases to 25°, it then returns, and disappearing in the morning in the light of the dawn, is soon after seen again in the evening, producing the same phenomena as before.

The extent of the greatest digressions of Mercury, or its greatest separation from the Sun on each side varies from * 18 to † 32 degrees.

The length of its entire oscillation or return to the same position relatively to the Sun, varies likewise from one hundred and six, to one hundred and thirty days, the mean arc of its retrogradation is about ‡ 15°, and its mean duration 23 days, but there is a great difference in these quantities in different retrogradations.

---

* 16° 12′. † 29. ‡ 13° 30′.

In general the laws of the motion of Mercury are extremely complicated, they do not take place exactly in the plane of the ecliptic; sometimes the planet departs * 5° from it.

A long series of observations was no doubt necessary to recognise the identity of the two stars, which were alternately seen in the morning and in the evening to depart from and return to the Sun; but as one never shewed itself till the other disappeared, it was at last suspected to be the same planet which thus oscillated on each side of the Sun.

The apparent diameter of Mercury is very variable, and its changes are evidently connected with its relative position to the Sun and the direction of his motion. It is a minimum either when the planet in a morning immerges into the solar rays, or when in the evening it disengages itself

---

* 4° 30′.

from them; it is at its maximum when it immerges into the Sun's light in an evening, or when it again becomes visible in the morning. Its mean apparent diameter is * 21″,3.

Sometimes, in the interval of its disapparition in the evening and its re-apparition in the morning, it is seen projected in the form of a black spot, upon the disk of the Sun on which it describes a chord.

It is known to be the same, by its position, its apparent diameter, and its retrograde motion being the same as it ought to be; these transits of Mercury are real annular eclipses of the Sun, from which we discover that the light of the planets is borrowed from it; when viewed in a good telescope it presents to us phases similar to those of the Moon, directed in the same manner towards the Sun, and of which the variable extent according to its re-

---

* 7″.

lative position with respect to it and following the direction of its motion, throws a great light on the nature of its orbit.

The planet Venus offers the same phenomena as Mercury, with this difference that its phases are much more sensible, its oscillations more extensive and their period more considerable. The greatest digressions of Venus vary from * 50° to † 53° and the mean length of its entire oscillations is 584^{d}. The retrogradations commence or finish when the planet approaching the Sun in the evening or receding from it in the morning is distant from it ‡32°. The mean arc of its retrogradation is about § 18°, and its mean duration forty-two days. Venus does not move exactly in the plane of the ecliptic, but sometimes deviates from it several degrees.

Like Mercury, Venus appears sometimes to describe a chord of the disk of the

---

* 45. † 47° 42'. ‡ 27° 48. § 16° 12.

Sun. The lengths of these transits over the Sun observed at different places on the Earth, remote from each other, are very sensibly different—this arises from the parallax of Venus, in virtue of which different observers refer it to different points of the solar disk, and see it describe different chords in its passage over it. In the transit which took place in 1769, the difference of its duration, as observed at Otaheite, in the South Sea, and at Lajanebourg in Swedish Lapland, amounted to more than * 15 minutes. As these durations can be calculated with great precision, their differences give very exactly the parallax of Venus, and consequently its distance from the Earth at the moment of conjunction. A remarkable law which we shall explain, at the end of the discoveries which made it known, connects this parallax with that of the Sun and planets; thus the observation of these

* 8′ 6″.

transits is of great importance in astronomy. After succeeding each other in the interval of eight years, they do not occur again for more than a century, when they succeed each other again, during an interval of eight years, and thus they continue.

The two last transits happened in 1761 and 1769. Astronomers were sent to different countries, where the observations could be made under the most favourable circumstances, and it is from the result of their observations that the parallax of the Sun has been concluded to be equal to * 27″2, at its mean distance from the Earth.

The great variations in the diameter of Venus prove that its distance is continually changing. The distance is the least at the moment of her transit over the disk of the Sun, its apparent diameter being then about † 177″. Its mean diameter is ‡ 51″ 54.

---

* 8′8. † 57″ 3. ‡16″ 6.

The motion of some spots upon its surface instructed Dominique Cassini that its rotation on its axis is performed in the interval of rather less than a day. Schroeter by continued observation of the variation of its horns and of some luminous points towards the edges of the unenlightened parts, has confirmed this result, concerning which some doubts had arisen. He has fixed the duration of its rotatory motion at † 0,973 days, and has found, like Cassini, that the equator of Venus makes a considerable angle with the ecliptic. Finally, he has concluded the existence of high mountains on its surface from his observations, and from the law by which its light gradually varies from its enlightened to its unenlightened side; he supposes the planet surrounded with an extensive atmosphere, the refracting power of which differs but little from that of the terrestrial atmosphere.

The extreme difficulty of seeing these spots even in the best telescopes, renders

---

* 34ʰ 15′ 43″2.

these observations almost impossible in our climates, but they merit the attention of observers, who, situated in a more southern latitude, enjoy a more favourable climate.

Venus surpasses in brightness all the other stars and planets; it is sometimes so brilliant as to be seen in full day and by the naked eye. This phenomenon, which is not unfrequent, never fails to excite surprise, and the credulous ignorance of the vulgar, suppose it connected with the remarkable events of the same time.

CHAP. VI.

Of Mars.

THE two planets which we have just considered seemto accompany the Sun like satellites,and their mean motion round the Earth is the same as that of the Sun. The other planets recede from the Sun to all possible angular distances, but their motions have such a connection with its position, as can leave no doubt of his influence on them.

Mars seems to move from west to east round the Earth; the mean length of a sidereal revolution is * $686^d\,979579$. Its motion is very unequal; when it begins to be visible in the morning it is direct and most rapid; it becomes gradually slower,and the planet when it arrives at about † 152° from the Sun is stationary; the motion then be-

---

* $1^y\,321^d\,23^h\,30'\,35''6$. † 136° 48'.

comes retrograde, increasing in velocity till the moment of opposition of the planet with the Sun. This velocity then becomes a *maximum*, diminishes, and again becomes nothing, when Mars, approaching the Sun, is distant from it 152°. Its motion then becomes again direct, after having been retrograde during 73 days, and in this interval the planet describes an arc of retrogradation of about * 18°, continuing to approach the Sun it finishes by immerging in the evening into its rays. These singular phenomena are renewed at every opposition of Mars, but with a considerable difference as to the extent and duration of these retrogradations.

Mars does not move exactly in the plane of the ecliptic, but deviates occasionally several degrees. The variations in its apparent diameter are very great. It is about † 30″, in its mean state, and augments to

* 16° 12′. † 13″ 3.

* 90″, as the planet approaches its opposition. At this time the parallax of Mars becomes sensible, and is nearly double that of the Sun. The same law which exists between the parallaxes of the Sun and Venus exists likewise between the Sun and Mars, and the observation of this last parallax had given a very near approximation of the solar parallax before the transit of Venus had ascertained it with greater precision.

The disk of Mars changes its form and becomes sensibly oval, according to the relative position of the Sun. These phases show that it is from the Sun it receives its light. From the observation of spots distinctly seen on its surface, it is inferred that it moves on itself from west to east in a period of † 1^d 02723, and on an axis inclined ‡ 66°. 33, to the ecliptic.

---

* 29″ 1. † 1^d 0^h 44″ 45″ 3. ‡ 59° 41′ 49″ 8.

CHAP. VII.

Of Jupiter and its Satellites.

JUPITER moves from west to east in a period of * 4332^{d} 602208. It is subject to inequalities similar to those of Mars previous to its opposition, and when it is nearly †128 distant from it, its motion becomes retrograde, its velocity augments till the moment of opposition, then diminishes, and recovers its usual direction, when the planet, in its approach towards the Sun is only 128° distant from it. The duration of this retrograde motion is about 121^{d} and the arc of retrogradation is ‡11°. But there are perceptible differences in the extent and duration of the retrograde

---

*11^{y} 318^{d} 14^{h} 27′ 10″ 7. † 115° 12′. ‡ 9° 54′.

motions of Jupiter. The path of this planet is not always confined to the ecliptic, it deviates from it occasionally *3 or 4 degrees.

Several obscure belts or stripes may be observed on the surface of Jupiter, evidently parallel among themselves, and also to the ecliptic; there are likewise other spots, the motion of which has demonstrated the rotation of this planet from west to east, upon an axis nearly perpendicular to the plane of the ecliptic, and in a period of †0d41377. The variations of some of these spots, and the sensible difference in the period of rotation, deduced from their motions, induce the opinion that they are not attached to Jupiter. They appear to be clouds which the winds transport with various velocities, in an extremely agitated atmosphere.

Jupiter is, next to Venus, the most bril-

---

* 2° 42′ or 3° 36′. † 9h 55′ 49″ 7.

liant of the planets, and sometimes even surpasses it in brightness. Its apparent diameter is the greatest possible; in its opposition, when it is *149″ its mean diameter in the direction of the equator, is †120″. But it is not equal in every direction. The planet is evidently flattened at the poles of its axis, and it has been found by very accurate measurement that its diameter in the direction of its poles is to that of its equator, nearly in the proportion of 13 to 14.

It is observed that there are four small stars round Jupiter, which incessantly accompany it. Their relative situation to each other changes every instant; they oscillate on each side of the planet, and it is by the extent of these oscillations that the rank of these satellites is determined, that being called the first satellite whose oscillation is the least. They are sometimes

---

* 48″2. † 38″ 8.

seen to pass over Jupiter's disk, and project a shadow, which then describes a chord of this disk. Jupiter and his satellites therefore are opaque bodies enlightened by the Sun, and when they interpose between the Sun and Jupiter, they produce real solar eclipses precisely similar to those which the Moon occasions on the Earth.

This phenomenon leads to the explanation of another which the satellites present. They are often observed to disappear, though at some distance from the disk of the planet; the third and fourth reappear sometimes, on the same side of the disk.

The shadow which Jupiter projects behind it (relatively to the Sun), is the only cause that can explain these disappearances, which are perfectly similar to eclipses of the Moon.

The circumstances which accompany them leave no doubt of the reality of this cause. The satellites are always observed to disappear on that side of the disk opposite to the Sun, and consequently on the

same side to which the conical shadow is projected. They are eclipsed nearest to the disk when the planet is nearest to its opposition. Finally, the duration of these eclipses answers precisely to the time which they should employ to traverse the cone of the shadow of Jupiter.

Thus these satellites move from west to east in returning orbits round this planet. Observations of their eclipses are the most exact means of determining their motions. Their mean sidereal and synodical revolutions as seen from the center of Jupiter, are very accurately determined by comparing eclipses at long intervals from each other, and observed near the opposition of the planet.

It is thus discovered that the motion of the satellites of Jupiter is nearly circular and uniform, since this hypothesis corresponds very nearly with those eclipses which happen when we see this planet in the same position relatively to the Sun.

Therefore the positions of the satellites

at every instant, as seen from the centre of Jupiter, may be determined. From hence results a simple and sufficiently exact method of comparing with each other the distances of Jupiter and of the Sun from the Earth. A method which the ancient astronomers did not possess. For the parallax of Jupiter is insensible even to the precision of modern observations when it is nearest to us. They only judged of its distance by the time of its revolution, as they estimated those planets to be the farthest from us whose period of revolution was the longest.

Let us suppose that the total duration of an eclipse of the third satellite has been observed. At the middle of the eclipse the satellite seen from the centre of Jupiter, is nearly in opposition to the Sun. Its sidereal position, observed from this centre, and which it is easy to conclude, from its mean motion, is therefore the same as that of the centre of Jupiter seen from that of the Sun.

Direct observation, or the known motion of the Sun, gives the position of the Earth as seen from its centre. Thus, imagining a triangle formed by the right lines which join the centres of the Sun ; the Earth, and Jupiter, we have in this triangle the angle of the Sun, observation will give that of the Earth, and we shall get at the instant of the middle of the eclipse the rectilinear distance from Jupiter to the Earth and to the Sun, in parts of the distance from the Sun to the Earth.

It is found by these means that Jupiter is at least five times farther from us than the Sun, when its apparent diameter is *120″. The diameter of the Earth at the same distance would not subtend an angle of †11″; the volume of Jupiter is therefore at least a thousand times greater than that of the Earth.

The apparent diameters of these satellites being insensible, their magnitudes can-

---

*38″ 3. † 3″ 5.

not be exactly measured. The attempt has been made to appreciate it by the time they take to penetrate the shadow of the planet. But there is a great discordance in the observations, which have been made to ascertain this circumstance; this arises from the various powers of telescopes, the different degrees of perfection in the sight of the observer, the state of the atmosphere, the altitude of the satellites above the horizon, their apparent distance from Jupiter, and the change of the hemisphere presented to us. The comparative brightness of the satellites is independent of the four first causes, which only alter their proportional light, and ought therefore to afford information concerning the rotatory motion of these bodies. Dr. Herschel who is occupied in this delicate investigation, has observed that they surpass each other alternately in brilliance, a circumstance that enables us tojudge of their respective light. The relation of the maximum and minimum of their light with their mutual po-

sitions, has persuaded him that they revolve upon themselves like a moon in the period equal to the duration of their revolution round Jupiter. A result which Maraldi had already concluded for the fourth satellite, from the returns of the same spot observed on its disk, in its passage over the planet. The great distance of the celestial bodies weakens the phenomena which their surfaces present, till they are reduced to slight variations of light, which escape the first view, and are only rendered sensible by a long course of observations. But this means of supplying the imperfection of our organs ought to be used with the greatest circumspection, to avoid being deceived concerning the causes on which these varieties depend.

CHAP. VIII.

Of Saturn, of its Satellites, and its Ring.

THE period of the sidereal motion of Saturn round the Earth, is *10759^{d}. 077215. This motion, which is from west to east, and nearly in the plane of the ecliptic, is subject to inequalities similar to those of Jupiter and Mars. It commences and finishes its retrograde motion when the planet before and after its opposition is about †121° distant from the Sun. The duration of this retrogradation is nearly 131 days, and the arc of retrogradation about ‡7°. At the moment of opposition, its diameter is a maximum, its mean magnitude is § 54″4. Saturn presents a phenomenon *unique* in the system of the universe. Two small bodies are always

---

* 29y 174^{d} 0^{h} 18′ 31″ 8. † 108° 54. ‡ 6° 18. § 17″ 6.

observed on each side of it, which appear to adhere to it, and of which the form and magnitude are extremely variable, sometimes they even disappear, and then Saturn appears round like the other planets. By following these singular appearances with care, and combining them with the positions of Saturn relatively to the Sun and the Earth, Huygens has discovered that they are produced by a large thin ring which surrounds the globe of Saturn and which is every where separated from it. This ring being inclined *54°8 to the plane of the ecliptic, always presents itself obliquely to the Earth, under the form of an ellipse, whose breadth, when at a maximum, is nearly the half of its length. In this position its shorter axis exceeds in diameter the disk of the planet. The ellipse becomes narrower in proportion as the visual ray, proceeding from Saturn to the Earth, becomes less inclined to the plane

* 49° 19′ 12″.

of the ring, the farthest arc of which is at length hid behind the planet. And the anterior arc is confounded with it. But its shadow is projected upon the disk, and forms an obscure band which can be seen in very powerful telescopes, which proves that Saturn and its ring are opake bodies enlightened by the Sun. No part of the ring can then be distinguished except that which projects on each side of Saturn; this diminishes by degrees, and finally disappears when the Earth in consequence of the motion of Saturn, is in the plane of the ring, the thickness of which is not perceptible. The ring disappears also when the Sun being opposite to it only enlightens its thickness. It continues invisible as long as it remains between the Sun and Earth, and reappears when both are on the same side the planet from the respective motions of Saturn and the Sun.

The plane of the ring meeting the solar orbit at every semi-revolution of Saturn, the phenomena of its disappearance and re-

appearance, return every fifteen years, but frequently with very different circumstances, two disappearances, and two re-appearances may occur in the same year, but never more. When the ring disappears its thickness reflects the light of the Sun to us, but in too small a quantity to be perceptible. Nevertheless it may be conceived, that it might be seen by augmenting the power of the telescope. Dr. Herschell found this in the last disappearance of Saturn's ring, which continued visible to him through the whole period, when it had ceased to be so to any other observer.

The inclination of the ring to the ecliptic is measured by the largest opening which the ellipse presents to us. The position of its nodes may be determined by the apparent situation of Saturn, when the ring disappears or reappears, the Earth being in its plane. All the disappearances and appearances from which the same sidereal positions of the nodes of the ring result, take place when its plane meets

the Earth. The others when the same plane meets the Sun. It may therefore be known by the situation of Saturn when the ring disappears or reappears, whether this phenomenon is produced by its plane meeting the Sun or the Earth. When this plane passes through the Sun, the position of its nodes gives that of Saturn, as seen from the centre of the Sun, and the rectilinear distance of Saturn from the Earth may be determined as that distance of Jupiter is by the eclipses of its satellites. It is thus found that Saturn is about nine times and a half farther from us than the Sun, when its apparent diameter is * 54″ 4. The apparent breadth of the ring is nearly equal to its distance from the surface of Saturn, both appear to be one-third the diameter of the planet. But on account of the irradiation its real breadth should be smaller.

Its surface is not uniform, a black band

---

* 17″ 6.

which is concentric with it, divides it into two parts, appearing to form two distinct rings. Observations of some bright points of this ring have proved to Dr. Herschel its rotation from west to east in a period of *0^{d} 437, round an axis perpendicular to its plane, and passing through the centre of Saturn.

Seven satellites have been observed in motion round this planet, from west to east in orbits nearly circular. The six first move nearly in the plane of the ring. The orbit of the seventh approaches more to the plane of the ecliptic. When this satellite is to the east of Saturn, its light becomes weaker till it is scarcely perceptible, which can only happen from the spots which cover the hemisphere which it presents to us. But for this phenomenon to occur always in the same position, it is requisite that this satellite (similar in that respect to our Moon and to the satellites of Jupiter) should revolve

* 10^{h} 29′ 16″ 8.

upon its own axis, in a period equal to that of its revolution round Saturn. Thus equality in the period of rotation and revolution appears to be a general law of the motion of satellites.

The diameters of Saturn, are not equal among themselves. That which is perpendicular to the plane of the ring, is less by at least one-eleventh than that situated in the plane of the ring. If this ellipticity be compared with that of Jupiter it may be concluded with great probability that Saturn turns rapidly round its shorter axis, and that the ring moves in the plane of the equator. Dr. Herschel has confirmed this result, by direct observation, he has found the duration of Saturn's rotation to be *0^d 428. This rotation is like all the motions of the planetary system from west to east. Dr. Herschel has likewise observed on the surface of the planet five belts nearly parallel to its equator.

---

† 10^h 16' 19" 5.

CHAP. IX.

Of Uranus and his Satellites.

The five planets that we have hitherto considered have been known from the most remote antiquity. The planet Uranus had escaped the observation of ancient astronomers from its minuteness. Flamstead at the end of the last century, and Mayer and Le Monnier in this, had observed it as a small star. But it was not till 1781 that Dr. Herschel discovered its motion, and soon after, by following this star carefully, it has been ascertained to be a true planet. Like Mars, Jupiter, and Saturn, Uranus moves from west to east round the Earth. The duration of its siderial revolution is *30689[d]. Its mo-

* 84[y] 29[d].

tion, which is nearly in the plane of the ecliptic, begins to be retrograde when, previous to the opposition, the planet is * 115° distant from the Sun. It ceases to be retrograde when, after the opposition, the planet in its approach to the Sun is only 115° distant from it. The duration of its retrogradation is about 151 days, and its arc of retrogradation, † 4 degrees. If the distance of Uranus were to be estimated by the slowness of its motion, it should be on the confines of the planetary system. Its apparent diameter is very small and hardly amounts to ‡ 12″. Dr. Herschel by means of a very powerful telescope, has discovered six satellites moving round this planet, in orbits almost circular and nearly perpendicular to the plane of the ecliptic.

---

* 103°30′. †3° 36″. ‡ 3″ 8.

CHAP. X.

Of Comets.

STARS are often observed, which, at first scarcely perceptible, augment in size and in velocity, then diminish, and finally disappear. These stars, which are called comets, appear almost always accompanied with a nebulosity, which increasing terminates sometimes in a train of considerable length, which is composed of some substance extremely thin, since the stars are seen through it. The appearance of comets, followed by these long trains of light, has for a long period terrified mankind, always agitated by extraordinary events of which the causes are unknown. The light of science has dissipated the vain terrors which comets, eclipses, and many other phenomena in-

spired in the ages of ignorance. The phase observed in the comet of 1744, of which only half the disk was enlightened, proves that these stars are opaque bodies which borrow their light from the Sun. Comets participate like all the other stars in the diurnal motion of the heavens, and this combined with the smallness of their parallax proves that they are not meteors engendered in our atmosphere. Their own motions are very complicated, and they are not confined like planets, in one direction, nor to planes little inclined to the ecliptic.

CHAP. XI.

Of the fixed Stars and their Motions.

THE parallax of the fixed stars is insensible; when examined by the most powerful telescopes, their disks are reduced to luminous points. It is in this circumstance that these celestial bodies differ from planets, the apparent diameter of which is increased by telescopes. The smallness of the apparent diameter of the fixed stars is proved particularly by their rapid disappearance in their occultations by the Moon, the time of which not amounting to one second, proves that their diameter is less than* five seconds of a degree.

---

* 1′ 6.

The vivacity of the light of the most brilliant stars, compared with the smallness of their apparent disk, leads us to believe that they are much farther from us than the planets, and that they do not like them borrow their light from the Sun, but are themselves luminous; and as the smallest stars are subject to the same motions as the most brilliant, and constantly preserve the same position relatively to each other, it is extremely probable that all these celestial objects are of the same nature, that they are so many luminous bodies of different magnitudes, and placed more or less distant from the planetary system.

Periodical variations have been observed in the intensity of the light of many of these stars, which have been called on that account changeable. Sometimes stars have been known to appear suddenly, and vanish after they have shone with the most brilliant splendour. Such was the famous star observed in 1572,

in the constellation of Cassiopeia; in a short time it surpassed the brilliance of the most beautiful stars, and even of Jupiter itself. Its light afterwards diminished, and it disappeared entirely sixteen months after its discovery, without having changed its place in the heavens; its colour suffered considerable alterations, it was at first of the most dazzling white, then of a reddish yellow, and finally of a lead-coloured white. What is the cause of these phenomena? The extensive spots which these fixed stars present to us periodically in turning on their own axis nearly in the same manner as the last satellite of Saturn, and the interposition of large opake bodies which circulate round them, are sufficient to explain the periodical variations of changeable stars. As to those stars which suddenly appear with a most brilliant light, and then vanish, it may be supposed with probability, that great conflagrations occasioned by extraordinary causes, take place on their

surface, and this supposition is confirmed by their change of colour, analogous to that which is presented to us on the earth by bodies which are consumed by fire.

A white light, of an irregular form, and to which the name of the Milky Way has been given, surrounds the heavens in the form of a zone. So great a number of small stars have been discovered in it by means of telescopes, that it is highly probable that the Milky Way is nothing more than an assemblage of these stars, which are so close together as to appear one continued band of light. Small white spots of light are also observed in different part of the heavens, which seem to be of the same nature as the Milky Way. Many of them viewed in a telescope equally display the union of several small fixed stars; others only present a white and continued light, perhaps on account of their great distance, which confounds the light of the stars of

which they are composed. These white spots are called nebulæ.

The constant position of the fixed stars with respect to each other, has determined astronomers to refer to them, as to so many fixed points, the motions proper to the other celestial bodies. But for this purpose it was necessary to class them, that they might be recognized; and it was with this design that the heavens have been divided into various groupes of stars called constellations. It was likewise necessary to ascertain with precision the position of the fixed stars on the celestial sphere, and it was thus accomplished: Let a great circle be conceived to pass through the two poles of the world, and through the centres or any celestial body; this circle is called the *circle of declination*, and cuts the equator perpendicularly: the arc of this circle, comprised between the equator, and the celestial object, measures its declination, which is *north* or *south* accord

ing to the denomination of that pole to which it is the nearest.

As all the heavenly bodies situated on the same parallel of the equator would have the same declination, it was necessary to assume a new element to determine their situation. The arc of the equator, contained between the circle of declination and the vernal equinox, has been chosen for this purpose. This arc, reckoned from the equinox in the direction of the motion proper to the Sun, that is, from west to east, is called the *right ascension*, thus the position of any celestial object is determined by its right ascension and declination. The meridian altitude of a celestial object compared with the latitude, gives its distance from the equator or its declination. The determination of its right ascension presented greater difficulties to the ancient astronomers on account of the impossibility of comparing directly the fixed stars with the Sun ; as the Moon might be compared

by day with the Sun, and by night with the stars, they employed it as a medium to measure the difference of right ascension of the Sun and the fixed stars; keeping in consideration the motions proper to the Moon and Sun during the interval of the observations. The theory of the Sun afterwards giving its right ascension, they obtained from it that of several of the principal fixed stars, to which they compared the others. It was by this means that Hipparchus formed the first catalogue of fixed stars of which we have any knowledge. Some time after, they gave greater precision to this method, by employing instead of the Moon the planet Venus, which can sometimes be perceived at noon, and the motion of which during a short interval of time is slower and more equal than the lunar motion.

At present, when the important application of the pendulum to clocks furnishes a very precise measure of time,

we can determine directly, and with an accuracy extremely superior to that of the ancient astronomers, the difference of right ascension between the Sun and any other celestial object, by the interval that elapses between their meridian transits. The position of the heavenly bodies may be referred in a similar manner to the ecliptic, which is principally useful in the theory of the Moon.

A great circle is supposed to pass through the centre of the object perpendicular to the plane of the ecliptic, and which is called a circle of latitude.

The arc of this circle included between the ecliptic and the star, measures its latitude, which is either north or south, according to the denomination of the pole situated on the same side of the ecliptic.

The arc of the ecliptic contained between the circle of latitude, and the vernal equinox, and reckoned from this equinox towards the east and west, is called the *longitude* of a celestial ob-

ject, the position of which is thus determined by its longitude and latitude. It may be easily conceived that the inclination of the ecliptic to the equator being known, the longitude and latitude of a star may be deduced from its observed right ascension and declination.

An interval of only a few years was necessary to observe the variation of the fixed stars in right ascension and declination. It was soon remarked that in changing their position relatively to the equator, they preserved the same latitude with regard to the ecliptic, and from hence it was concluded that their variations in right ascension and declination were owing to the motion of the celestial sphere round the poles of the ecliptic. These variations might also be represented by supposing the fixed stars immovable, and making the poles of the equator move round those of the ecliptic. In this motion the inclination of the equator to the ecliptic remains the same,

and its nodes or the equinoxes retrograde uniformly about * 154″ 63 annually. It has been shewn previously that this retrogradation of the equinoxes renders the tropical year a little shorter than the sidereal year.

Thus the difference of the tropical and sidereal years, and the variation of the fixed stars in right ascension and declination, depends on this motion by which the pole of the equator describes annually an arc of 154″ 63, of a small circle of the celestial sphere parallel to the ecliptic. It is in this that the phenomenon consists, known by the name of the precession of the equinoxes.

The precision of modern astronomy, for which it is principally indebted to the application of telescopes, to astronomical instruments, and that of the pendulum to clocks, has rendered minute periodical inequalities perceptible in the inclination

---

* 50′ 1.

of the equator to the ecliptic, and in the precession of the equinoxes. Bradley who discovered them, and who has followed them with great attention during many years, has discovered their law, which may be represented in the following manner:—Let the pole of the equator be supposed moved upon the circumference of a small ellipse, tangent to the celestial sphere, whose centre, which may be regarded as the mean pole of the equator, describes uniformly every year 154″63 of the parallel of the ecliptic on which it is situated. The greater axis of this ellipse, always tangent to the circle of latitude, and in the plane of this great circle, subtends an angle of about * 62″ 2, and the smaller axis subtends an angle of 46″ 3. The situation of the real pole of the equator upon this ellipse is determined thus:—there is supposed to be a small circle on the plane of the ellipse, concentric with it, and the diameter of

* 20″ 1. † 14″ 9.

which is equal to its greater axis; conceive a radius of this circle, moved uniformly with a retrograde motion, so that this radius coincides with that half of the greater axis nearest to the ecliptic, every time that the mean ascending node of the lunar orbit coincides with the vernal equinox. Finally, from the extremity of this moveable radius, suppose a perpendicular let fall upon the greater axis of the ellipse. The point where this perpendicular cuts the circumference of this ellipse, is the place of the real pole of the equator; this motion of the pole is called its nutation. The fixed stars, in consequence of the motions we have described, preserve among themselves a constant position. But the illustrious observer to whom we owe the discovery of the nutation, has found in all the fixed stars, a general and periodical motion, which alters a little their respective positions; to represent this motion, it must be imagined that each star describes

annually a small circumference parallel to the ecliptic, the centre of which is the mean position of the fixed star, and the diameter of which, seen from the Earth, subtends an angle of * 125″ ; and that it moves upon this circumference, like the Sun in its orbit, in such a manner that the Sun may be always more advanced than the star by † 100° ; this circumference, projected upon the surface of the heavens, appears under the form of an ellipse, more or less flattened, according to the height of the star above the ecliptic ; the lesser axis of this ellipse being to the greater axis as the sine of this altitude is to radius ; from hence arise all the varieties of this periodical motion of the fixed stars, which is called aberration.

Independent of these general motions, many fixed stars have movements peculiar to themselves ; very slow, but which the

---

* 40″ 5. † 90°.

lapse of time has rendered sensible, and which give us reason to believe that all the fixed stars have similar motions, which will develope themselves in succeeding ages. It has been hitherto principally remarkable in Syrius and Arcturus, two of the most brilliant of the fixed stars.

CHAP. XII.

Of the Figure of the Earth, and of the Variation of Gravity on its Surface; and of the Decimal System of Weights and Measures.

LET us now descend from the heavens to the earth, and see what can be derived from observations relative to its dimensions and figure. It has been already seen that the earth is nearly spherical; gravity every where directed towards its centre, retains bodies on its surface; though in places directly opposite to each other, they are in contrary positions. The sky and stars always appear above the earth, for elevation and depression are only relative terms regarding the direction of gravity.

From the moment that man recognized

the spherical form of the globe which he inhabits, his curiosity would lead him to measure its dimensions. It is then extremely probable that the first attempts to attain this object were made at a period much anterior to those of which history has preserved the record, and that they have been lost in the physical and moral changes which the earth has experienced; the relation which many measures of the most remote antiquity have to each other, and to terrestrial circumference, strengthens this conjecture, and seems to indicate not only that this measure was very accurately known at an extremely ancient period, but that it has served as the base of a complete system of measures, the vestiges of which have been found in Egypt and Asia. Be that as it may, the first precise measure of the earth, of which we have any certain knowledge is that which Picard executed in France, towards the end of the last century, and which has been many

times verified ; it is easy to imagine this operation. As we advance towards the north, the pole seems to be more and more elevated, the meridian altitude of stars situated towards the north augments, and that of the southern stars diminishes, some even become invisible. The first idea of the curvature of the earth was owing no doubt to the observation of these phenomena, which could not fail to attract the attention of mankind in those first ages of society when the return of the seasons was only distinguished by the rising and setting of the principal stars, compared with that of the Sun. The elevation or depression of the stars, gives the angle which verticals elevated at the extremity of the arc passed over, form at their point of contact ; for this angle is evidently equal to the difference of the meridian altitudes of the same star, minus the angle which the arc described, would subtend at the centre of the star ; and we are certain that

this is insensible. It was then only requisite to measure this arc. It would be a long and tedious operation to apply our measures to so great an extent; it is a much more simple process to connect its extremities by a chain of triangles, to those of a base of 12 or 15,000 feet, and considering the precision with which the angles of these triangles may be determined, its length can be obtained very exactly. It is thus that the arc of the terrestrial meridian which crosses France from Dunkirk to Montjoui near Barcelona has been measured; that part of this arc, whose amplitude is equal to the hundredth part of a right angle, and whose central point corresponds to * 51°¼ of latitude, is equal to 100,179 metres.

Of all the re-entering figures, the spherical one is the most simple, since it only depends on a single element, the size of its radius. The natural inclination of the human mind, to attribute that form to bodies

* 46° 10′ 12″.

which it comprehends with the greatest facility, disposed it to give the Earth a spherical form. But the simplicity of nature should not always be measured by that of our conceptions. Infinitely varied in her effects, nature is only simple in her causes, and her œconomy consists in producing a great number of phenomena, often very complicated, by means of a small number of general laws.

The figure of the Earth is the result of these laws, which, modified by a thousand circumstances, might alter it sensibly from a sphere. Small variations observed in the length of the degrees in France indicate these alterations. But the inevitable errors of observation left doubts on this interesting phenomenon, and the academy of sciences, in which this great question was anxiously agitated, judged with reason that the difference of the terrestrial degree, if it really existed, would be principally manifested in the comparison of the degrees at the equator, and towards the poles. And acade-

micians were sent even to the equator, where they found the degree of the meridian equal to 99552^{me}. 3, less by 465^{me}. 6 than the degree corresponding to the mean parallel. Other academicians travelled towards the north to about *75°,7 of latitude, and the degree of the meridian was observed to be 100696^{me}. 0 greater by 1143^{me}. 7 than at the equator; thus the increase of the meridional degrees from the equator to the poles was incontestably proved by these measures, and it was discovered that the earth was not exactly spherical.

These celebrated journies of the French academicians, having directed the attention of astronomers towards this object, new degrees of the meridian were measured in Italy, Germany, Africa and Pennsylvania. All these measures concurred to indicate an increase of the degrees from the equator towards the poles. The ellipse being, next to the circle, the most simple of

* 66° 19′ 48″.

the re-entering curves, the earth was considered as a solid formed by the revolution of an ellipse about its shorter axis. Its compression in the direction of the poles is a necessary inference from the observed increase of the meridional degrees from the equator to the poles.

The radii of these degrees, being in the direction of gravity, are, by the law of the equilibrium of fluids, perpendicular to the surface of the sea, with which the earth is in a great measure covered. They do not tend as in a sphere to the centre of the ellipsoid. They have neither the same direction nor the same length as radii drawn from this centre to the surface, and which cut it obliquely every where but at the equator and at the poles. The point of contact of two adjoining verticals, is the centre of the small terrestrial arc which they comprise between them; if this arc was a straight line, these verticals would be parallel, or would only meet at an infinite distance, but in proportion as they are curved, they meet at a dis-

tance so much the shorter as they are more curved. Thus the extremity of the shorter axis being the point where the ellipse approaches most to a straight line, the radius of a degree at the pole, and consequently the degree itself, is of its greatest length. It is the contrary at the extremity of the greater axis of the ellipse. At the equator where the curvature is the greatest, the degree in the direction of the meridian is the shortest. Passing from the second to the first of these extremes the degrees augment, and if the ellipse is but little flattened their increase is very nearly proportional to the square of the sine of the latitude.

The measure of two degrees in the direction of the meridian is sufficient to determine the two axes of the generating ellipse, and consequently the figure of the Earth, supposing it elliptic. If this is the hypothesis of nature the same proportion should be found very nearly between the two axes, comparing two by two the degrees of France, of the north, and of the equator;

but their comparison gives in this respect differences which it is difficult to attribute to errors of observation alone. The excess of the axis of the equator above that of the pole taken as unity is called the compression or ellipticity of the elliptic spheroid; now the degrees of the north and of France give $\frac{1}{146}$ for the ellipticity of the Earth, which the degrees of France and at the equator give $\frac{1}{334}$, it appears then that the Earth differs sensibly from an ellipsoid. There is even reason to believe that it is not a solid of revolution, and that its two hemispheres are not equal on each side of the equator. The degree measured by La Caille at the Cape of Good Hope in the southern latitude of *37° 01 has been found to be 100050^{me}. 5, which surpasses the degree in France in the latitude of †51°$\frac{1}{3}$, and is greater than that which has been measured in Pennsylvania, in the latitude of ‡ 43° 56, the length of which was only 99789^{me}. 1.

---

* 33° 18′ 32″ 9. † 46° 11′ 49. ‡ 39° 12′ 14″ 4.

The degree at the Cape is yet greater than the degree measured in Italy in the latitude of * 47° 80, and which has been found to be 99948^{me}. 7. Yet it ought to be smaller than all these degrees if the earth was a regular solid of revolution, formed of two similar hemispheres. Every thing leads us to believe that this is not the case. Let us see what then is the nature of the terrestrial meridians.

The celestial meridian as determined by astronomical observations, is formed by a plane which passes through the centre of the earth and the zenith of the observer, because this plane bisects the arcs of the parallels to the equator, which the stars describe above the horizon.

All the places on the Earth which have their zeniths in the circumference of this meridian form the corresponding terrestrial meridian. Considering the immense distance of the stars, verticals elevated from

---

* 43° 37′ 12′.

each of these places may be considered parallel to the plane of the celestial meridian. The terrestrial meridian may then be defined a curve formed by the junction of the bases of all the verticals parallel to the plane of the terrestrial meridian. This curve lies altogether in this plane when the Earth is considered as a solid of revolution, in every other case it departs from it, and generally is one of those lines which geometricians call *curves of double curvature.*

The terrestrial meridian is not exactly the line which determines trigonometrical measurements in the direction of the celestial meridian. The first side of the line measured is a tangent to the surface of the earth, and parallel to the plane of the celestial meridian.

If this side be prolonged till it meets a vertical infinitely near it, and this prolongation be bent to the base of the vertical, the second side of the curve will be formed, and thus with all the others. The line

thus traced is the shortest that can be drawn on the surface of the earth between any two points taken on this line. It is not in the plane of the celestial meridian, and is not confounded with the terrestrial meridian, except in the case where the Earth is a solid of revolution; but the difference between the length of this line, and that of the corresponding arc of the terrestrial meridian is so small that it may be neglected without any sensible error.

The figure of the Earth being extremely complicated, it is important to multiply the measures of it in every direction, and in as many places as possible.

We may always, at every point of its surface, suppose an osculatory ellipse, which sensibly coincides with it, to a small extent round the point of contact.

Terrestrial arcs measured in the direction of the meridian, and perpendicular to it, compared with observations of latitudes, and of the angles which the direction of the

extremities of these arcs form with their respective meridians, will inform us of the nature and position of this ellipsoid, which may not be a solid of revolution, and which varies sensibly at great distances.

The operations which Delambre and Mechain have executed in France to obtain the length of the metre, determine very nearly the osculatory ellipse of that part of the terrestrial surface. They have observed the latitude not only at the extremities of the arc, but at three intermediate points.

The astronomical and trigonometrical observations were made with a repeating circle, which gives great precision in the measure of angles.

Two bases of above twelve thousand metres have been measured, the one near Melun, the other near Perpignan. And what confirms the correctness of all these observations is, that the base at Perpignan deduced from that at Melun, by the chain

of triangles which unites them, does not differ by the third of a metre from its measurement, though the distance which separates the two bases is above nine hundred thousand metres.

The principal results of this important operation are shewn in the following table:

Observed Latitudes.

	Decimal.	Sexigonal.
Montjoui -	45°. 958281	41° 21' 45''
Carcassone -	48. 016790	41 12 54
Evaux -	51. 309414	46 10 42
Pantheon at Paris	54. 274614	48 50 49.7
Dunkirk -	56. 706944	51 29 10

Arc of the terrestrial meridian comprised between Montjoui and

Carcassone - -	205621mo 3
Evaux - - -	534714 5
Pantheon - -	831536 4
Dunkirk - -	1075058 5

The comparison of these results evidently indicates a diminution in the terrestrial degrees from the pole to the equator, but

the law of this diminution seems very irregular. Notwithstanding if the ellipsoid which satisfies these measures nearer than any other be required, we find that to represent them in this hypothesis, it is sufficient to alter the observed latitudes about four seconds and a half.

The compression is then $\frac{1}{150}$, the semi-axis of the pole parallel to that of the Earth, is 6344011^{me}, and the degree corresponding to the mean parallel is 99983^{me}. 7. An error of four seconds and a half, though very small, is not admissible, considering the great precision of the observations; but we may at least consider this ellipsoid as osculatory to the surface of the earth in France, at * 51° of latitude, and suppose that it coincides with it to an extent of 5 or 6 degrees round the point of osculation. It gives 100716^{me}. 9, for the degree perpendicular to the meridian, at † 56° 3144 latitude, and by a very exact operation lately made in England it has been found to be 100700^{me}. 5.

* 45° 54'. † 50° 40' 58".

This agreement proves that the action of the Pyrenees and other mountains in the south of France has had very little influence on the latitudes observed at Evaux, Carcassone and Montjoui, and that the great compression of the osculatory ellipse depends on attractions much more extended, the effect of which is felt in the north as well as the south of France, and even in England, Italy, and Austria; for the degrees which have been carefully measured are very nearly the same as on this ellipsoid. There is reason then to presume that if the arc measured from Dunkirk to Montjoui was extended to the island of Cabrera, the corresponding degree of the mean parallel which would result from this measure would not exceed one hundred thousand metres. The total arc included between this island and Dunkirk, being divided into two nearly equal parts by this parallel; the length of the quarter of the meridian concluded from this arc, becomes totally independant of any hypothesis of the ellipticity of the Earth.

Mechain had consequently proposed to join Cabrera to Montjoui, and had already prepared every thing for this new measure; but events have not suffered him to perform it: let us hope that favorable circumstances will soon permit it to be resumed.

It appears by the observed directions of the sides of the arc measured from Dunkirk to Montjoui, that the osculatory ellipse is not exactly a solid of revolution. But more certain information would be obtained on this subject, if, as is much to be desired, a perpendicular to the meridian of the observatory was to be measured in the broadest part of France with the same means that were employed in the measure of the meridian, and if the latitude and direction of its sides relative to their respective meridians were to be determined with precision on different points of this perpendicular.

Whatever be the nature of the terrestrial meridians, it is evident as their degrees diminish from the poles to the equator, that the Earth is flattened in the direction of

its poles, that is to say, that the axis of the poles is less than the diameter of the equator. To explain this, let us suppose the Earth a solid of revolution, and let us represent the radius of the degree at the north pole, and the series of all these radii from the pole to the equator, which radii by the supposition continually diminish. It is visible that these radii form by their consecutive intersections a curve which at first touches the axis of the pole, it afterwards separates from it, its convexity being constantly turned towards it, and raises itself towards the pole till the radius of the meridional degree takes a direction perpendicular to the first: it will then be in the plane of the equator. If this radius of the polar degree be supposed flexible, and that it involves successively the arc of the curve which we have considered, its extremity will describe the terrestrial meridian, and the part of it intercepted between the meridian and the curve, will be the corresponding radius of the degree of the me-

ridian. This curve is what geometricians call the evolute of the meridian. Let us consider at present, the intersection of the diameter of the equator with the axis of the pole, as the centre of the Earth. The sum of the two tangents to the evolute of the meridian drawn from this centre, the one following the axis of the pole and the other the diameter of the equator, will be greater than the arc of the evolute which they include between them. Now the radius drawn from the centre of the Earth to the north pole, is equal to the radius of the polar degree, *minus* the first tangent; the semi-diameter of the equator is equal to the radius of the degree of the meridian at the equator, *plus* the second tangent. The excess of the semi-diameter of the equator above the terrestrial radius of the pole is then equal to the sum of these two tangents *minus* the excess of the radius of the polar degree, above the radius of the degree of the meridian at the equator; this last excess is the arc itself of the evolute,

which is less than the sum of the extreme tangents. The excess then of the semi-diameter of the equator above the radius drawn from the centre of the Earth to the north pole is positive. It can be proved in the same manner, that the excess of the semi-diameter of the equator above the radius drawn from the centres of the earth to the south pole is positive. The whole axis of the poles thus is less than the diameter of the equator, or, what comes to the same thing, the Earth is flattened in the direction of the poles.

Considering every portion of the meridian as a very small arc of its osculatory circumference, it is easy to see that the radius drawn from the centre of the Earth to that extremity of the arc which is nearest to the pole, is less than the radius drawn from the same centre to the other extremity. From whence it follows, that the terrestrial radii increase from the poles to the equator, if, as all observations indicate, the degrees of the meridian augment

from the equator to the poles. The difference of the radii of the degrees of the meridian from the pole to the equator is equal to the difference of the corresponding terrestrial radii, *plus* the excess of twice the evolute above the sum of the two extreme tangents, which excess is evidently positive: thus the degrees of the meridian increase from the equator to the poles in a greater proportion than the diminution of the terrestrial radii. It is evident that these demonstrations equally apply, if the two northern and southern hemispheres were not equal and similar, and it is easy to extend them to the supposition of the earth's not being a solid of revolution.

But it is remarkable that the observations made in the northern hemisphere give the evolute of the meridian from * forty-three to † seventy-three degrees of latitude very little different from that of an ellipsoid of $\frac{1}{150}$ compression, and of which the mean degree is 99983$^{me.}$ 7. For this

* 38° 42'. † 65° 42'.

ellipsoid nearly satisfies the measures lately made in France, the degrees measured in Italy and Lapland, and that which has been measured in England perpendicular to the meridian.

It also represents the degree of the meridian measured in Austria at * 53° of latitude, and which Liesganig has found to be 100114$^{mc.}$ 2. Finally it agrees with the degree of the longitude measured in France at † 48°4 latitude, and of which Cassini and La Caille have fixed the length at 72003 $^{mc.}$ 5.

Curves have been constructed at the principal places in France on the line which has been considered as the meridian of the observatory of Paris, traced in the same manner as this line, with this difference only that the first side, always a tangent to the surface of the Earth, instead of being parallel to the plane of the celestial meridian of Paris, is perpendicular to it.

---

* 47° 47′ 24″ † 43° 33′ 36″

It is by the length of these curves and by the distance from the observatory to the points where they meet the meridian, that the position of these places is determined. This labour, the most useful to geography which has yet been performed, is a model which every enlightened nation will no doubt, hasten to imitate.

To connect objects thus, it is requisite that they should be but little distant from each other. To fix the respective positions of places separated by great distances or by the sea, recourse must be had to celestial observations. The knowledge of these positions is one of the greatest advantages which astronomy has produced. To arrive at it, the method has been followed that was made use of to form the catalogue of stars, conceiving on the terrestrial surface circles similar to those which had been imagined on the surface of the heavens.

Thus the axis of the celestial equator intersects the surface of the earth in two

points diametrically opposite, which have each one of the poles of the world for their zenith, and which may be considered as the poles of the earth.

The intersection of the plane of the celestial equator with this surface, is a circumference which may be regarded as the terrestrial equator. The intersection of all the planes of the celestial meridian with the same surface, form so many curved lines, which unite at the poles, and which are the corresponding terrestrial meridians, if the earth is a solid of revolution, which may be supposed in geography without any sensible error. Finally, small circles traced from the equator to the poles upon the earth parallel to the equator, are terrestrial parallels, and that of any place whatever, corresponds to the celestial parallel which passes through its zenith.

The position of a place upon the earth is determined by its distance from the equator, or the arc comprised between its pa-

rallel and the equator ; and by the angle which its meridian forms with a principal meridian, the position of which is arbitrary, and to which all the others are referred. The distance from the equator depends on the angle included between the zenith and the celestial equator, and this angle is evidently equal to the altitude of the pole above the horizon. This altitude, is called latitude in geography. Longitude is the angle which the meridian of a place makes with the principal meridian; it is the arc of the equator contained between these two meridians, it is east or west according as the place is east or west of the principal meridian. An observation of the altitude of the pole gives the latitude. The longitude is determined by means of any celestial phenomenon observed at the same time on the meridians of which the respective position is required. The instant of noon is not the same on these meridians ; if that from which the longitude is reckoned is to the east of that of which the longitude is

required, the Sun will arrive sooner at the celestial meridian.

If, for example, the angle formed by the terrestrial meridians be a quarter of the circumference, the difference between the instants of noon on these meridians will be a quarter of a day. Suppose then, that a phenomenon is observed on each of them, which occurs at the same physical moment to every place on the Earth, such as the beginning or end of an eclipse of the moon or of the satellites of Jupiter. The difference of time which the observers will reckon at the moment of the phenomenon will be to the whole day as the angle formed by the two meridians is to the circumference.

Eclipses of the Sun, and occultations of stars by the Moon, furnish the most exact means of finding longitudes, from the precision with which the beginning and end of these phenomena may be observed. They do not in fact occur at the same physical instant to the whole earth, but the elements of the lunar motions are sufficient-

ly known to enable us to keep an exact account of this difference. To determine the longitude of a place, it is not necessary that the celestial phenomenon should be observed at the same time on the principal meridian; it is sufficient if it is observed on one whose position with the principal meridian is known. It is thus that by connecting the meridians with each other the position of the most distant points on the surface of the Earth is ascertained. The longitudes and latitudes of a great number of places on the Earth are already known by astronomical observation. Great errors in the position and extent of countries long known have been corrected. And the situation of new countries which the interest of commerce and the love of science have caused to be discovered, is fixed.

But though voyages undertaken in these later periods have added considerably to our geographical knowledge, much yet remains to be discovered. The interior of Africa and America include vast countries

which are totally unknown. We have only uncertain and frequently contradictory relations concerning many others, of which, geography, hitherto abandoned to the hazard of conjecture, only waits for new information from astronomy, to determine the positions irrevocably.

It is principally to the navigator, when in the midst of the ocean with no other guide then the stars and his compass, that it is important to know his own position, that of the places for which his course is bound, and of the rocks and quicksands which may occur in his passage.

He may easily know his latitude, by observations of the stars; but the sphere, in consequence of its diurnal motion, presenting itself daily in nearly the same manner to all the points of his parallel, it is difficult for the navigator to fix the point on which he is. To supply the deficiency of celestial observations, he measures his velocity and the direction of his motion, he estimates his passage in the

direction of the parallels, and comparing it with his observed latitude, determines his longitude relat ive to the place of his departure. The inaccuracy of this method exposes him to errors, which might become fatal when he abandons himself during the night to the winds, near shores or banks, which from estimation he believed himself far from. It is to secure him from these dangers that as soon as the progress of arts and astronomy led to the hope that methods might be found to obtain the longitude at sea, commercial nations have hastened to direct the views of scientific men and artists to this important object by powerful encouragements.

Their wishes have been fulfilled by the invention of chronometers, and by the accuracy to which the tables of the lunar motions have been carried. Two methods, good in themselves, and which are yet improved by lending each other a mutual support.

A chronometer, well regulated in a

port the situation of which is known, and preserving the same rate, when carried on board a vessel, would indicate at every instant the time which was reckoned in this port. Comparing it with that observed at sea, the relation of the difference of these hours to the whole day, would be, as has been already seen, that of the corresponding difference of longitude to the whole circumference. But it was difficult to obtain such watches, the irregular motion of the vessel, the variations of temperature, and the inevitable friction which is very sensible in so delicate a machine, were so many obstacles which opposed their accuracy.

These have been fortunately overcome, and chronometers are now made, which, during many months, preserve a rate nearly uniform, and thus afford the most simple method of finding the longitude at sea; and as this method is so much the more precise as the time is short, during which these chronometers are employed without

verifying their rate, they are very useful to determine the respective position of places very near to each other. They have in this respect, some advantage over astronomical observations, the precision of which is not increased by the smallness of the observer's distance.

The eclipses of the satellites of Jupiter, which occur frequently, offer to a navigator an easy method of knowing his longitude, if he could observe them at sea; but the endeavours that have been made to overcome the difficulties arising from the motion of the vessel, which oppose this species of observation, have hitherto been fruitless. Navigation and geography have however derived great advantages from these eclipses, and chiefly from those of the first satellite, of which the commencement and end can be observed with precision. The navigator employs them with success when he can land. It is necessary indeed to know the hour at which the same eclipse that he observes would

be seen upon a known meridian, since the difference of time which is reckoned on these two meridians, gives the difference of longitude ; but the tables of the first satellite of Jupiter, considerably improved in our time, give the instants of these eclipses, with a precision almost equal to observation.

The extreme difficulty of observing these eclipses at sea, has compelled us to have recourse to other celestial phenomena, among which the lunar motions are the only ones which can be made subservient to the determination of terrestrial longitudes. The position of the Moon, such as it would be observed at the centre of the earth, may be easily concluded from the measure of its angular distances from the Sun and stars. The tables of its motion then give the hour at the principal meridian, when the same phenomenon is observed, and the navigator, comparing the time which he reckons on board his vessel, at the moment of observation, de-

termines his longitude by the difference of time.

To appreciate the accuracy of this method, it should be considered that from the errors of observation, the Moon's place, as determined by the observer, does not correspond exactly to the hour pointed out by his chronometer, and that from the errors of the tables, this same place does not relate to the corresponding hour, which they indicate on the first meridian; the difference of these hours then would not be that given by an observation and tables rigorously correct. Let us suppose that the error produced by this difference is a minute. In this interval forty minutes of the equator pass the meridian; this is the corresponding error in the longitude of the vessel, and which, upon the equator, is about forty thousand metres, but it is less on the parallels: besides, it may be diminished by multiplying the observations of the lunar distances from the Sun and stars, and repeating them during

many days, to compensate and destroy by each other, the errors of the observations and tables.

It is obvious that the errors of longitude corresponding to those of the tables and of observation, are so much the less as the motion of the celestial body is more rapid; thus the observations of the Moon in perigee, are in this respect preferable to those in apogee. If the motion of the Sun be employed, which is about thirteen times slower than that of the Moon, the errors of longitude will be about thirteen times as great: from whence it follows, that of all the celestial bodies, the Moon is the only one whose motion is sufficiently rapid to be employed for the determination of longitudes at sea. It is evident therefore how important it is to render these tables perfect.

A very remarkable phenomenon, the knowledge of which we owe to astronomical voyages, is the variation of gravity at the surface of the earth. This va-

gular power acts in the same spot, upon all bodies proportionally to their masses, and tends to impress on them equal velocities in equal times. It is impossible, by means of a balance, to ascertain these variations, because they affect equally the body which is weighed, and the weight to which it is compared ; but observations of the pendulum are those most proper to discover them, since it is clear that these oscillations should be slower in those places where gravity is least powerful. This instrument, the application of which to clocks is one of the principal causes of the progress of modern astronomy, consists of a body suspended at the end of a thread or a rod, moveable round a fixed point, placed at the other extremity. The instrument is drawn a little from its vertical position, then abandoning it to the action of gravity, it makes small oscillations, which are very nearly of the same duration, notwithstanding the difference of the arcs described. This duration de-

pends on the size and figure of the suspended body, on the mass and length of the rod; but geometricians have found general rules to determine, by the observed oscillations of a compound pendulum of any figure whatever, the length of a pendulum, whose oscillation shall have a certain duration, and in which the mass of the rod shall be supposed nothing relative to that of the body, considered as an infinitely dense point. It is by this imaginary pendulum, called the *simple pendulum*, that all the experiments of the pendulum, made in different parts of the Earth, are referred.

Richer, sent in 1672, to Cayenne, by the academy of sciences, to make astronomical observations there, found that his clocks, regulated at Paris to mean time, lost a perceptible quantity every day.

This interesting observation gave the first direct proof of the diminution of gravity at the equator. It has been repeated with care, in a great number of

places, keeping account of the resistance and temperature of the air.

The result of all the observed measures of a pendulum vibrating seconds is, that it increases from the equator to the poles, like the degrees of the meridian, and that its increase, which, at the pole itself, is equal to five hundred and fifty-five hundred thousandths of its gravity at the equator, is proportional to the square of the sine of the latitude.

Borda, by a very exact experiment, has found recently that the length of the pendulum vibrating seconds at the Observatory at Paris, and reduced to a vacuum, is $0^{me}.741887$; it results that its length in France, under the parallel of * 50° is equal $0^{me}.741606$, and that thus the simple pendulum of the length of the metre would make 86116.5 oscillations in a day.

This length, which is very exact, and the measure of the degree of the meri-

* 45°.

dian, corresponding to the same parallel, will serve to recover our weights and measures, if by the lapse of time they should alter.

There has likewise been remarked, by means of the pendulum, a minute diminution of gravity on the summits of high mountains. Bouguer has made a great number of experiments on this subject at Peru. He has found that the force of gravity at the equator, and at the level of the sea being expressed by unity, is 0.999249 at Quito, elevated 2857 metres above this level; and 0.998816 on the summit of Pichenca, at 4744^{me} in height. This diminution of gravity at altitudes always very small relatively to the terrestrial radius, gives us reason to suspect that this force diminishes considerably at great distances from the centre of the Earth.

I ought, when speaking of the observations of the pendulum, to call the attention of natural philosophers to the two following circumstances: One is, the slight

resistance which bodies, in changing their temperature, appear to me to oppose to their change of volume, nearly as water resists its conversion into ice, and can retain its form at several degrees below zero. It is then sufficient to agitate it to render it solid. In the numerous experiments on the dilatation of bodies, which I made with Lavoisier, we were sometimes obliged to give them a slight concussion to make them take the form proper to their temperature. The second object relates to the invariable pendulums which are used to determine the difference of gravity on various places of the earth. If the rod of the pendulum is of steel, its to be feared that the effect of terrestrial magnetism should become complicated with that of gravity; and as the object is to appreciate very small quantities in these experiments, it is important to be assured that this effect is insensible.

Observations of the pendulum vibrating seconds, furnishing a length which is in-

variable and easily recoverable at any period, have given rise to the idea of employing an universal measure.

The prodigious number of measures in use, not only among different people, but in the same nation ; their whimsical divisions, inconvenient for calculation, and the difficulty of knowing and comparing them; finally, the embarrassment and frauds which result from them in commerce, cannot be observed without acknowledging that the adoption of a system of measures, of which the uniform divisions are most easily subjected to calculation, and which are derived in a manner the least arbitrary, from a fundamental measure, indicated by nature itself, would be one of the greatest services which science and government can render to the human race. A people who should adopt such a system of measures, would unite to the advantage of gathering the first fruits of it, that of seeing their example followed by other nations, of which it would thus become the benefactor. For the

slow but irresistible empire of reason, at length overcomes all the national jealousies and obstacles which oppose themselves to an advantage that would be universally felt.

Such were the reasons that determined the constituent assembly to charge the academy of sciences with this important object. The new system of weights and measures is the result of the labors of a commission appointed by them, and aided by the zeal and abilities of many members of the national representation.

The identity of the decimal calculus with that of integral numbers, leaves no doubt of the advantages of dividing every kind of measure into decimal parts.

To be convinced of this, it is sufficient to compare the difficulty of complicated multiplication and division, with the facility of the same operations on integral numbers, which facility may be still increased by logarithms, the use of which might be made extremely popular by

means of instruments very simple, and of little expense. The academy, therefore, did not hesitate to adopt the decimal division; and to render the whole system of measures uniform, it was resolved that they should all be derived from the same linear measure, and its decimal divisions. The question was thus reduced to the choice of this universal measure, to which the name of *metre* was given.

The length of the pendulum, and that of the meridian, are the two principal means which nature offers to fix the unity of linear measures. Both independent of moral revolutions, cannot experience a sensible alteration, but by great changes in the physical state of the world. The first expedient, easy in use, has this inconvenience, of making the measure of distance depend on two heterogeneous elements, gravity and time, the measure of which is besides arbitrary. The second measure was therefore chosen, which appeared to have been employed in the most

remote antiquity: so natural is it to man to compare itinerary measures even to the dimensions of the globe which he inhabits, that in travelling he may know by the mere denomination of the space he has passed over, the relation of that space to the whole circuit of the earth. There is likewise the advantage of making nautical and celestial measures correspond. It is often necessary to the navigator to determine one by the other, the distance he has traversed, and the celestial arc included between the places of his departure and arrival. It is then interesting that one of these measures should be the expression of the other by nearly the difference of their unities. But for this purpose the fundamental unity of linear measures should be an aliquot part of the terrestrial meridian, which corresponds to one of the divisions of its circumference; thus the choice of the metre was reduced to that of the unity of angles.

The right angle is the limit of the in-

clination of a line to a plane, and of the altitude of objects above the horizon. Besides, it is in the first quarter of the circle that sines are formed, and, generally, all the lines employed in trigonometry, and the proportion of which to the radius, are reduced to tables. It is then natural to take the right angle for the unity of angles, and the quarter of the circumference for the unity of their measures. It is divided into decimal measures, and that the measures on the earth might correspond, the quarter of the terrestrial meridian is divided into the same parts, which had been done at a very ancient period; for the measure of the earth, mentioned by Aristotle, and the origin of which is unknown, gives a hundred thousand stadia to the quarter of the meridian. It was then only requisite to obtain its exact length. Here many questions present themselves, which the ignorance we are in respecting the true figure of the earth will not permit us to resolve. Is

the earth a spheroid of revolution? Are its two hemispheres equal and similar on each side of the equator? What is the proportion of an arc of the meridian, measured at a given latitude, to the whole meridian? In the most natural hypothesis of the formation of the terrestrial spheroid, the difference of the meridians is insensible; and the decimal degree, bisected by the mean parallel between the north pole and the equator, is the hundredth part of the quarter of the meridian. The error of these hypotheses, if any exists, cannot influence geographical distances, where it is of no importance. The length of the quarter of the meridian may then be concluded from that of the arc which crosses France, from Dunkirk to the Pyrenées, and which has been carefully measured in 1740, by the French academicians. But as a new measure of a greater arc, in which still more precise methods were employed, would inspire an interest in favor of the new system of measures, likely to

extend the use of it, it was resolved to measure the arc of the terrestrial meridian contained between Dunkirk and Barcelona.

The operations which Delambre and Mechain have lately performed, have given this arc, the amplitude of which is * 10° 748663, equal to fifty-five million one hundred and fifty eight thousand four hundred and seventy hundredths of the iron toise used at the equator at the temperature, † 16°¼. The middle of the arc being ‡ one degree and a third, more north than the mean parallel, the quarter of the meridian could not be determined by this measure, without adopting an hypothesis of the ellipticity of the Earth: that which results from the arc measured in France compared with that in Peru, appeared to deserve the preference, from the length and distance of the two arcs, and from the care and reputation of the observers. The

* 6° 40′ 26″ 2. † 14° 37′ 35″. ‡ 1° 11′ 24″.

quarter of the meridian was thus found to be equal to 5130740 toises.

The ten thousandth part of this length was taken for the metre, or unity of the linear measures. The decimal above this was too large, that below too small; and the metre, the length of which is 0,513074 toises, supplies the place advantageously of the toise and ell, the two of our measures in most common use.

To preserve the length of the metre, the national convention has decreed, that a standard executed from the experiments and observation of the committee charged with its determination, should be preserved in the care of the legislative body.

This standard, which was presented by the national institute, can only represent the metre, at one determined degree of temperature. That of melting ice has been chosen as the one most fixed and independent of the modifications of the atmosphere. To recover the metre at any period without being obliged to have recourse to the

measure of the great arc from which it was derived, it was important to fix very precisely its proportion to the length of the pendulum vibrating seconds; and it is with this view that Borda has again determined this length at the observatory at Paris.

All the measures are derived from the metre in the most simple manner; the linear measures are decimal multiples and submultiples of it.

The unity of the superficial measures of land is a square the side of which is ten metres, it is called *are*.

A measure equal to a cubic metre and usually employed to measure fire wood, is called *stere*.

The unity of cubic measure is the cube of the tenth of a metre, it is called *litre*.

The unity of weight which is called *gramme*, is the absolute weight of a cube of the hundredth part of a metre of distilled water, and considered at its *maximum* of density.

By a remarkable singularity this *max-*

imum does not correspond to the freezing point, but is above it at about four degrees of the thermometer. Water in falling below this temperature again dilates, and thus prepares itself for that increase of volume which it receives in its passage from the fluid to the solid state. Water was chosen as one of the most homogenous substances, and which can be most easily reduced to a state of purity. Le Fevre Gineau has determined the *gramme* by a long series of delicate experiments on the specific gravity of a hollow cylinder of brass, the volume of which he has measured with extreme care. The result is, that the livre, supposed the twenty-fifth part of the pile of fifty *marcs*, which is preserved at the mint of Paris, is to the *gramme* in the proportion of 489,5058 to unity.

All these measures being constantly compared with the *livre* in money it was particularly important to divide it into decimal parts. It is now called, *the silver*

franc, its tenth part, *decime*, its hundredth, *centime*.

To facilitate the calculation of the fine gold and silver contained in pieces of money, the alloy has been fixed at a tenth of their weight, and that of the *franc* has been made equal to five *grammes*. Thus the pieces of money are exact multiples of the unity of weight, which is extremely useful in commerce.

Finally, the uniformity of the whole system of weights and measures seemed to require that the day should be divided into ten hours, the hour into one hundred minutes, the minutes into a hundred seconds, &c. This division of the day which will become necessary to astronomers, is less advantageous in civil life, where there is little occasion to employ time as a multiplier or divisor. The difficulty of adapting it to clocks and watches, and our commercial connexions with foreigners in the sale of watches have suspended its use indefinitely. We may however believe that

at length the decimal division of the day will supersede its actual division, which differs too much from the division of other measures not to be abandoned.

Such is the new system of weights and measures which the men of science have offered to the national convention, which has hastened to sanction them. This system, founded on the measure of terrestrial meridians, is equally suitable to every nation. It has no other relation with France than by the arc of the meridian which crosses it; but the position of this arc bisected by the mean parallel, and the extremities of which are bounded by the two oceans, is so advantageous that the learned of every nation, united to fix an universal measure, would have chosen it.

To multiply the advantages of this system, and to render it useful to the whole world, the French government has invited foreign powers to take part in an object of such general interest. Many have sent eminent men of science to Paris, who uni-

ted with the committee of the national institute, have determined the fundamental unities of weight and length, by the discussion of observations and experiments; so that the determination of these unities ought to be considered as a work common to the learned who have assembled there, and the people whom they represent. It is then permitted to hope that this system, which reduces all measures and their calculation to the scale and to the most simple operations of decimal arithmetic, will be as generally adopted as the system of numeration to which it is analogous, and which without doubt had to oppose the same obstacles, which prejudices and habit oppose to the introduction of the new measures.

CHAP. XIII.

Of the Flux and Reflux of the Ocean.

Although the earth and the fluids which cover it must long since have assumed the state which is suitable to the equibrium of the forces which animate them, nevertheless the figure of the ocean changes every instant of the day, by regular and periodical oscillations, which are known by the name of tides. It is a circumstance truly astonishing, to behold even in calm and serene weather, the intense agitation of this great fluid mass, whose waves constantly break with impetuosity on the shore.

This phenomenon gives rise to reflection, and excites a strong desire to penetrate the cause. But not to be bewildered in vague hypothesis, we should first know the laws

of this singular appearance and follow them in their details.

At the commencement of this century, by the request of the Academy of Sciences, a great number of observations were made in our ports of the flux and reflux of the ocean.

They were continued every day at Brest during six consecutive years, and they form from their number, and from the magnitude and regularity of the tides at this port, the most valuable and complete collection of observations of this kind that we possess. Since a thousand accidental causes may alter the course of nature in these phenomena, it is necessary to consider at once a great number of observations, that the partial causes, compensating each other, the mean results may show only the effects, which are regular and constant. It is likewise necessary, by a happy combination of observations, to make those phenomena evident which we wish to determine, and by separating them from the rest, to make them

better known. It is from thus investigating these observations, that I arrived at the following result, of the truth of which there can remain no doubt.

The ocean rises and falls twice in every interval of time comprehended between two consecutive returns of the Moon to the superior meridian. The mean interval of these returns is* 1,035050 days; thus the mean interval of two consecutive high tides is † 0, 51525 of a day, so that there are some solar days in which we may observe but one single tide. The moment of low water divides this interval nearly into equal parts, nevertheless at Brest the tide employs ‡ nine or ten minutes more to descend than to rise. Similar to all magnitudes which are susceptible of a maximum and a minimum, the increase and diminution of the tides near the limits are proportional to the squares of the time elaps-

---

* $1^d\ 0^h\ 49'\ 16''\ 2$. † $12^h\ 15'\ 14'\ 1$.
‡ $12'\ 57''\ 6$ or $14'\ 24''$.

ed since the moments of high and low water.

The elevation of the water at high tide is not constantly the same ; it varies every day and its variation is évidently connected with the phases of the Moon. It is greatest about the time of the new and full moons, then it diminishes and becomes least about the quadratures. At Brest the highest tide does not take place exactly on the day of the sysygy, but a day and a half later ; so that if the sysygy happens at the moment of high water the greatest tide is the third that follows. In the same manner if the quadrature happens at the moment of high tide, the third tide which follows is the least. This phenomenon is observed to be nearly the same in all the ports of France, but the hours of high and low water are very different. The greater the elevation of the water at high tide, the greater the depression at the low tide which succeeds it. We call a *total tide* the half sum of the heights of two consecutive high tides

above the level of the intermediate low tide. The mean value of this total tide at Brest is $5^{me.}$ 888 at its maximum about the sysygies, and $2^{me.}$ 789 at its minimum about the quadratures.

The distance of the Moon from the Earth influences in a very perceptible manner, the heights of the tides. All other circumstances being the same, they augment and diminish with the lunar parallax but in a greater ratio. If this parallax increases $\frac{1}{16}$, the total tide increases $\frac{1}{8}$ in the sysygies, and about $\frac{1}{4}$ in the quadratures; and as the tide is nearly twice as great in the first as in the second case, its increase in the two cases is the same.

The greatest variation in the diameter of the Moon being (either more or less,) about $\frac{1}{15}$ of the whole, the corresponding variation of the total tide in the sysygies is $\frac{3}{40}$ of its mean height, or about $0^{me.}$ 88,3: thus the entire effect of the change of distance between the Moon and the Earth is $1^{me.}$ 766, in the total tides.

The variation in the distance of the Sun

from the Earth, influences the tides in a similar manner, but in a much less degree. All other circumstances being the same; in the winter when the Sun is nearest to us, the tides of the sysygy are the greatest, and those of the quadrature less than in summer, when the Sun is farthest from the Earth.

The declinations of the Sun and Moon have a remarkable influence on the tides, they diminish the total tides of the sysygies, and the tides at Brest are about $0^{mc}.8$ less in the solstices than at the equinoxes; the total tides of the quadratures are likewise less by the same quantity in the equinoxes than at the solstices.

It is principally about the maxima and minima of the total tides, that it is interesting to know the law of the variation. We have seen that the moment of the maximum at Brest follows the sysygy a day and a half. The diminution of those total tides that are near it, is proportional to the squares of the times elapsed from that instant to the time of the inter-

mediate low tide, to which the total tide is referred, it is $0^{me.}1064$, when this interval is a lunar day. Near the instant of the minimum which follows the quadrature one day and a half, it is proportional to the square of the time elapsed since that instant; it is nearly double the diminution of the tides near their maximum.

The declinations of the Sun and of the Moon influence very sensibly these variations. The diminution of the tides at the sysigies of the solstices is not above three fifths of the corresponding diminution of the sysygies of the equinoxes. A small difference has likewise been observed between the tides of the morning and the evening, which depends on the declinations of the Sun and Moon, and which disappears when these bodies are in the equator. This variation may be perceived, by comparing the tides of the first and second day after the sysygy or the quadrature, because the tides being then very near their maximum or minimum, vary very little from one day to another, and the more easily admit the difference to

be perceived, between two tides in the same day. Thus it is found at Brest that at the sysygy of the summer solstice the morning tides the first and second day after it, are less than the evening tides by $0^{me.}183$. They are on the contrary greater by the same quantity in the sysygy of the winter solstice. In like manner in the quadratures of the autumnal equinox the morning tides the first and second day after the quadrature surpass the evening tides by $0^{me.}131$; they are less by the same quantity in the quadratures of the vernal equinox.

Such are in general the phenomena that are observed in our ports relatively to the height of our tides. Their intervals offer other phenomena which we are now to develope. When the high tide happens at Brest the moment of the sysygy, it follows the instant of mid-night, or mid-day * $0^{d}.14822$, according as it happens in the morning or in the evening: this interval which is very different in harbours that are even

* 3 h. 33′ 33″. 6.

very near each other, is called the hour of the port, because it is from that hour that we reckon in determining the times of high water relatively to the phases of the Moon, as will be explained in treating of the causes of the tides. The high tide that takes place at Brest the moment of the quadrature follows the instant of mid-night or mid-day *0^{d} 35464. The tide of the sysygy advances or retards †264″ for every hour that it precedes or follows the sysygy; the tide of the quadrature advances or retards ‡ 416″ for every hour that it precedes or follows the quadrature. The hours of high water at the sysygies and the quadratures vary with the distances of the Sun and Moon from the Earth and principally with the distances of the Moon. In the sysygies, every minute of increase or diminution in the apparent semi-diameter of the Moon advances or retards the hour of high water by §354″. This phenomenon likewise takes place at the quadratures but is three times less.

---

* 9^{h} 61′ 40″ 8. † 1′ 26″ 1. ‡ 2′ 20″ 7. § 1′ 54″ 6.

In a similar manner the declinations of the Sun and Moon influence the time of high water at the sysygies and the quadratures. In the sysygies of the solstices the time of high water advances about * 2 minutes, it is retarded by the same quantity in the sysygies of the equinoxes; on the contrary on the quadratures of the equinoxes the time of high water advances about † 8 minutes, and retards about the same quantity in the quadratures of the solstices.

We have seen that the retardation of the tides from one day to another is ‡ $0^d.03505$ in its mean state; so that if the time of high water happens at $0^d.1$, it will happen the next morning at § 0.13505. But this retardation varies with the phases of the Moon; it is the smallest possible about the sysygies, when the total tides are at their maximum, and then it is only ¶ $0^d.02705$; when the tides are at their minimum it is the greatest possible, and amounts to ‖ $0^d.05207$. Thus the difference of the times of high

* $2' 52'' 8.$ † $11' 31'' 2.$ ‡ $50' 28'' 2.$ § $2^h 24''.$
¶ $38' 57'' 1.$ ‖ $1^h 14'' 58'' 8.$

water corresponding to the moment of the sysygy and quadrature (and which is shewn above to be *0^{d}20642) increases for the tides which follow in the same manner these two phases, and become nearly equal to a quarter of a day relatively to the maximum and minimum of the tides.

The variations in the distances of the Sun and Moon, and principally of the latter, influence the retardation of the tides from one day to the other. Every minute of increase or of diminution in the apparent semi-diameter of the Moon, augments or diminishes this retardation † 258″ about the sysygies. This phenomenon takes place likewise in the quadratures, but is three times less.

The daily retardation of the tides varies likewise with the declinations of the Sun and Moon. In the sysygies of the solstices it is about ‡ 155″ greater than in its mean state; it is less by the same quantity in the sysygies of the equinoxes: on the contrary

---

* 5^{h} 21′ 14″ 6. † 3′ 42″ 9. ‡ 2′ 49″.

in the quadratures of the equinoxes, it surpasses the mean by * 543″, and is surpassed by the same quantity in the quadratures of the solstices. Thus the inequalities in the heights and in the intervals of the tides, have very different periods, some of half a day, and a day, others of half a month, a month, a half year, and of a year; others again have the same periods with the revolutions of the nodes, and of the perigee of the lunar orbit; the position of which influences the tides by the effect of the declinations of the Moon, and of its distances from the Earth.

The height, and generally all the phenomena of the tides, appear to have been the same in the new as in the full Moon.

These phenomena equally take place, in all the harbours and coasts along the shores of the sea; but the local circumstances, without changing at all the above laws of the tides, influence very materially the heights of the tides, and the hour of high water for a given port.

---

* 7′ 49.

CHAP. XIV.

Of the Terrestrial Atmosphere and of Astronomical Refractions.

A THIN, transparent, and elastic fluid envelopes the Earth, and extends itself to a considerable altitude; it gravitates like all other bodies, and its weight balances that of the mercury in the barometer.

At the temperature of melting ice and at the mean height of the barometer at the level of the sea, which height is nearly seventy-six *centimètres*, the weight of air is to that of a similar column of mercury in the proportion of unity to 10283. Therefore at this temperature to lower the mercury a centimetre when its height is seventy-six, it is sufficient to raise the barometer 102.83 metres, and if the density of the atmosphere was every where the same, its height would be 7815 metres. But the air

is compressible very nearly in the ratio of the weight which presses on it; from whence it follows that at equal temperatures its density is proportional to the height of the barometer; its inferior strata are therefore more dense than the superior ones, being compressed by their weight.

They become rarer in proportion as we elevate ourselves in the atmosphere; and if their temperature were the same a very simple calculation would shew us, that their altitude, increasing in arithmetical progression, their density would diminish in geometrical progression.

The cold which exists in the elevated regions of the atmosphere augments the density of the higher strata: for air, like all other bodies, contracts by cold, and expands by heat; and it has been observed that towards the temperature of melting ice the increase of a degree in its temperature augments its volume about a 250th part.

We have taken advantage of these data to measure the heights of mountains by means of the barometer. If at every pe-

riod and throughout its whole extent, the temperature of the atmosphere was every where equal to that of melting ice, the result would be, that multiplying by 17972$^{mc.}$ 1, the tabular logarithm of the proportion of the height of the barometer observed at any two stations, we should obtain the height of one of the stations above the other. But this height requires a correction relative to the erroneous hypothesis of a uniform heat and temperature equal to zero. It is obvious that if the mean temperature of the stratum of air contained between the two stations is greater than zero, its density diminishes and we must ascend, to make the barometer fall the same quantity. The multiplier 17972$^{mc.}$ 1, must then be augmented by as many times its 250th part, as there are degrees in this mean temperature, which is the same as observing the degrees of the thermometer at the two stations, and multiplying their sum by 35$^{mc.}$ 944, to add the product to 1792$^{mc.}$ 1. A slight correction should likewise be applied to the height of the barometer on ac-

count of the difference of temperature at the two stations. The density of the mercury is not the same in both.

Now its dilatation relative to the increase of a degree in its temperature is a 5412th part of its volume; the height of the barometer in the coldest station must then be augmented by its 5412th part, taken as many times as there are degrees of difference in the temperature of the two stations.

By means of this rule we shall obtain very nearly the difference of their heights, if they separate but little from the same vertical.

The air is invisible in small masses, but the rays of light reflected by all the strata of the atmosphere produce a sensible impression; they give it a blue shade, which spreads a tint of the same colour over all distant objects and which forms the celestial azure. This blue vault to which the stars appear to be attached, is then very near us, it is only the terrestrial atmosphere, beyond which these bodies are placed at immense distances. The solar rays,

which its particles transmit to us in abundance before the rising and after the setting of the Sun, produce the dawn and twilight, which extending to above * 20° distance from that luminary, proves to us that the extreme particles of the atmosphere are elevated at least sixty thousand metres.

If the eye could distinguish and refer to their true place the points of the exterior surface of the atmosphere, we should see the heavens like the segment of a sphere formed by the portion of this surface which would be cut off by a plane tangent to the surface of the Earth. And as the height of the atmosphere is very small in proportion to the terrestrial radius, the sky would appear under the form of a flattened vault. But though we cannot distinguish the limits of the atmosphere, yet as the rays which it transmits come from a greater depth of the horizon than the zenith, we ought to consider it more extended in the first direction. To this cause is joined the

---

* 18°.

interposition of objects in the horizon, which contributes to augment the apparent distance of that part of the heavens which we refer to it, the sky, therefore, should appear to us very much flattened, like a small portion of a large sphere.

A star at * 26° of altitude, seems to divide into two equal parts the length of the curve, which the section of the surface of the heavens by a vertical plane, forms from the horizon to the zenith. From whence it follows that the horizontal radius of this apparent celestial vault is to its vertical radius nearly as three and a quarter is to unity, but this proportion varies with the causes of this illusion.

The magnitude of the Sun and Moon being proportional to the angle under which they are seen, and to the apparent distance of that point of the heavens to which they are referred, they appear larger at the horizon, though they are seen under a smaller angle.

---

* 23° 24'.

The rays of light do not move in a straight line through the atmosphere, they inflect continually towards the Earth. The observer, who perceives objects in the direction of the tangent of the curve which they describe, sees them more elevated than they really are, and the stars appear above the horizon even when they are sunk below it. Thus the atmosphere, by reflecting the rays of the Sun, lengthens our enjoyment of its presence, and augments the duration of the days, which are still further prolonged by the dawn and twilight.

It is extremely important to astronomers to determine the laws and quantity of refraction, to obtain the true position of the stars. But before I present the result of their researches on this object, I will explain in a few words the principal properties of light.

A luminous ray in passing from one transparent medium to another approaches or recedes from the perpendicular to the surface which separates them. The law of refraction is such that, *the sines of the*

angles which its directions form with this perpendicular, the one before, the other after its entrance into the new medium, are in a constant ratio whatever the angles may be. But light when refracted presents a remarkable phenomenon which has led to the discovery of its nature. A ray of white light, received in a dark chamber, after its passage through a prism, forms an oblong image variously coloured, this ray is a pencil of an infinite number of rays differently coloured, which the prism separates in consequence of their various refrangibility. The most refrangible ray is the violet, then the indigo, blue, green, yellow, orange and red : but though we only distinguish seven rays, the continuation of the image proves that there exists an infinity which approach each other by insensible shades of refrangibility and colour. All these rays, collected by means of a lens, produce the white colour, which is therefore only a mixture of all the simple or homogeneous colours, in determined proportions.

When a ray of a perfectly homogeneous

colour is well separated from the others, it does not change either its refrangibility or colour, whatever reflections and refractions it may undergo.

It is not then due to the modification which light receives in the media which it traverses, but it is a natural property. Nevertheless a similitude of colour does not prove similitude of light; by mixing together several differently coloured rays of the solar image decomposed by the prism, we may form a colour perfectly similar to one of the simple colours of this image; thus the mixture of homogeneous red and yellow, produces an orange, similar in appearance to the homogeneous orange, but the refraction of the combined rays through another prism separates them, and makes the component colours reappear, while the rays of the homogeneous orange remain unaltered.

Rays of light are reflected if they fall on a mirror, making with the perpendicular to its surface, angles of reflection equal to the angles of incidence.

The refractions and reflections which rays of light undergo in drops of rain produce the rainbow, the explanation of which, founded on a rigorous calculation which satisfies exactly all the details of this curious phenomenon, is one of the most beautiful results of natural philosophy.

Most bodies decompose the light which they receive, they absorb one part and reflect the other in every direction; they appear blue, red, green, &c. according to the colour of the rays which they receive in the greatest abundance. Thus the white light of the Sun diffusing itself over all nature is decompesed and reflects to our eyes an infinite variety of colours.

After this short digression on light I return to astronomical refractions. Very precise experiments have ascertained that at the same temperature the refractive power of air increases or diminishes as its density. But at equal densities does this force vary as the temperature? What is the influence of the hygrometrical state of the

air, and of the proportion in which the two gases azote and oxygen are combined in the atmosphere, upon refraction? It is this which is unknown, and which, considering the importance of the object, deserves to be investigated.

Till the present time it has been supposed that the refractive power of the atmosphere only depends on the density of its strata; so that to determine the path of light which traverses it, it was sufficient to know the law of their temperature. But this law is unknown to us, and besides it varies at every instant. The temperature of the atmosphere being supposed every where the same, and equal to that of melting ice, the density of its strata diminishes in geometrical progression, and the refraction is *74′ in the horizon, it would only be † 56′, if the density of the atmospherical strata diminished in arithmetical progression, and became nothing at its surface. The horizontal refraction which is observed to

---

*39′ 57″ 6. † 30′ 14″ 4.

be about * $64'\frac{1}{2}$, is a mean between these limits. The law of the diminution of the density of the strata is then nearly the medium between the arithmetical and geometrical progressions, which agrees with the observations of the barometer and thermometer. In general all these observations and those of astronomical refraction may be reconciled by means of very probable hypotheses of the diminution of heat from ascending in the atmosphere without having recourse, as some natural philosophers have done, to a particular fluid which, mingled with the air, produces refraction.

When the apparent altitude of a star exceeds † 12°, the refraction only depends sensibly on the state of the barometer and thermometer in the place of the observer; and it is nearly proportional to the tangent of the apparent distance of the star from the zenith diminished by four times the refraction. It has been discovered by different

---

* 34′ 49″ 8. † 10° 48′.

means that at the temperature of melting ice, when the height of the barometer is 76 centimetres, the co-efficient, which multiplied by this tangent gives the astronomical refraction is * 185″.9, but it varies as the density of the air in the place of observation: this density varies a 250th part for every degree of the thermometer; this co-efficient must therefore be increased or diminished as many times its 250th part, as the thermometer of the observer indicates degrees above or below zero; the density of the air at unequal temperatures being proportional to the height of the barometer, the co-efficient thus corrected must be varied in the proportion of the observed altitude of the barometer, to 76 centimetres.

By means of these data, a table of refractions may be constructed from 1½° of apparent altitude to the zenith, in which interval almost all astronomical observations are made.

This table would have the advantage of being independent of all hypotheses of the constitution of the atmosphere, and might be as useful at the summits of the highest mountains as at the level of the sea.

The atmosphere weakens the light of the stars, particularly at the horizon where their rays traverse it through a greater extent.

It follows from the experiments of Bouguer, that the barometer being at 76 centimetres in height, if we take the intensity of the light of a star at its entrance into the atmosphere as unity, the intensity of a star in the zenith when it reaches the observer is reduced to 0,8125.

The height of the atmosphere reduced throughout its whole extent to the density of the air corresponding to zero of temperature, and to the pressure of a column of mercury $0^{me.}\cdot 76$ in height, would be $7815^{me.}$ Now it is natural to suppose that the extinction of a ray of light which traverses it is the same as in this hypothesis, since it meets the same number of aerial particles: thus a stratum of air of the preceding density and

of 7815$^{me.}$ thickness, reduces the force of light to 0 8123.

It is easy to conclude from hence the diminution of light, in a stratum of air of equal density and of any given thickness; for it is obvious that if the intensity of light be reduced to one quarter in traversing a given thickness, an equal thickness will reduce this quarter to a sixteenth of its original value; from whence we see that as the thickness increases in arithmetical progression, the intensity of the light diminishes in geometrical progression: its logarithms then follow the ratio of the thickness: thus, to obtain the tabular logarithm of the intensity of light, when it crosses a stratum of air of a given thickness, we must multiply 0.0902835, the logarithm of 0.8123, by the proportion of this thickness to 7815$^{me.}$ and if the density of the air is greater or less than the preceding, this logarithm must be augmented or diminished in the same proportion.

To determine the diminution of the light of stars relating to their apparent altitude

we may imagine a luminous ray moving in a channel every where of the same size, and we may reduce the air in this channel to the preceding density. The length of the column of air thus reduced will determine the diminution of the light of a star thus considered. Now we may suppose from the zenith to about 12° of apparent altitude, the strata of the atmosphere, sensibly plane and parallel, and the passage of light rectiliniar, then the thickness of every stratum in the direction of the luminous ray is to its depth in a vertical direction as the secant of the apparent distance of the star from the zenith is to radius; multiplying then this secant by 0.0902835, and by the proportion of the height of the barometer to $0^{me.}$ 76, we shall get the logarithm of the intensity of the stars' light. This very simple rule will give the diminution of the light of the stars, at the summits of mountains and at the level of the sea, which may be useful, both to correct the eclipses of Jupiter's satellites, and to estimate the intensity of the solar light at the focus of

burning glasses. We ought, however, to observe that the vapors floating in the air influence considerably the extinction of the light of stars; the serenity of the heavens in southern climates makes their light in general much more brilliant there; and if we were to transport our great telescopes to the top of the high mountains of Peru, there is no doubt but we should discover many celestial phenomena, which a thicker and less transparent atmosphere renders invisible in our climates.

The intensity of light at small altitudes depends on the constitution of the atmosphere as well as refraction. If its temperature was every where the same the logarithms of the intensity of light would be proportional to the astronomical refractions, divided by the cosines of the apparent altitude, and then this intensity at the horizon would be reduced to the four thousandth part of its primitive value: it is on this account that the Sun, whose lustre at noon is too dazzling to be borne, can be regarded without pain in the horizon.

It is natural to think that every particle of the Sun's surface sends in every direction the same quantity of light. Two equal and very small portions of this surface, seen from the Earth, one at the centre of the disk and the other towards its edge, appear to occupy different spaces, which are to each other as the radius is to the cosine of the arc of the great circle of the solar surface which separates these two parts, therefore the intensity of the light is in an inverse ratio. Notwithstanding this Bouguer has found by experiments that the light of the Sun is more brilliant at the centre than towards the edges. In comparing that of the centre, with the light of a point distant from the limb by a quarter of the semi-diameter, the intensities of these two lights appear to be in the proportion of 48 to 35. This difference indicating a thick atmosphere round the Sun which weakens its light, it follows from the preceding results, and from the experiment of Bouguer, that the intensity of the light of a star seen from the surface of the Sun, at the zenith,

is reduced to 0.24065, and that the Sun deprived of its atmosphere, would appear twelve times and a third more luminous. A horizontal stratum of air, at the temperature of zero, and under the pressure of a column of mercury $0^{me.}76$ ought to have $53548^{me.}$ of thickness to weaken light in the same degree as the atmosphere of the Sun.

This then would be the height of this atmosphere reduced to the density of this aerial stratum, if at equal densities its transparency was the same as that of the air, but of this we are ignorant. However, these results are subordinate to the correctness of Bouguer's experiment, which deserves to be repeated with care, in the different aspects of the solar disk. The vibrations of the air produce sounds which, according to the rapidity or slowness of the vibrations, are sharp or grave. But whatever their nature may be, the rapidity of their propagation is the same, and sound strong or weak, grave or sharp, traverses $2914^{me.}$ in a second.

The winds, from the zephyr to the most

impetuous whirlwinds, are produced by the air being displaced with more or less violence. In the most violent tempests this velocity is about thirty metres in a second. It is only about a third of this in ordinary winds. Without doubt, the cause which raises regularly the waters of the sea, and which appears to reside in the Sun and Moon, disturbs equally the equilibrium of the atmosphere, which it must penetrate to act upon the ocean. But the periodical winds which result from it are too weak to be observed amidst the agitations which the atmosphere experiences from a great number of other causes.

It is in the atmosphere that cloud, storms, the aurora borealis, and all meteors are formed. The air dissolves water, and this dissolving property varies with its density and its heat.

Thus, water is dissolved and precipitated alternately in the atmosphere, in consequence of all the causes which vary the temperature and density of the air. The

water of the sea in dissolving, abandons the salt which it contains. It descends under the form of dew, snow, rain, or hail, a part of which, collected by mountains and elevated places, is filtrated by the earth it passes through, and forms springs and rivers which return to the sea.

Electricity obtains a passage through the atmosphere with difficulty: its different strata are habitually electrified, and this appears to increase in proportion to their height. The clouds formed in the higher strata are then more electrified than the lower strata into which they descend. But whatever be the cause of the electricity of clouds, it is asserted that thunder is an electric explosion between the clouds and the earth.

Air is not a homogeneous substance; experiments have proved that it is composed of three parts of a gas called azote, and on part of oxygen gas, a gas peculiarly respirable, in which bodies exhibit a brilliant light during combustion. It is this gas alone

which is necessary to combustion and to the respiration of animals, which is known to be a slow combustion and the principal source of animal heat.

Other aëriform fluids mingle with the atmosphere and rise in it, in proportion to their specific lightness. The lightest of these fluids is that which is called hydrogen gas, it is 15 or 16 times lighter in its state of purity than the atmospheric air. Combined with oxygen gas in nearly the proportion of one to six, it forms water, which far from being an element, as was believed for a long time, may be composed and decomposed at pleasure. The decomposition of bodies in marshes and stagnant water, developes a great quantity of hydrogen gas which is carried to the confines of the atmosphere, where being enflamed by natural electricity, it produces falling stars, globes of fire, and those trains of light which are observed in great heats, and which being seen sometimes at the same instant, at great distances, indicate that their height is at least one hundred thousand metres.

When contained in some light substance, hydrogen gas rises with the bodies which are attached to it till it meets with a stratum of the atmosphere, sufficiently rare for it to rest in equilibrium.

By these means, the fortunate discovery of which we owe to the French philosophers, man has extended his domain and his power. He may launch into the air, traverse the clouds, and interrogate nature in the exalted regions of the atmosphere formerly inaccessible.

The atmosphere transmits the light of the Sun freely, and heat with difficulty. It increases therefore the heat at the surface of the Earth; and perhaps without the resistance which it opposes to the diffusion of the solar heat, we should experience excessive cold at the equator itself.

It is to heat that the aëriform state of the atmosphere is owing; it is the pressure of the atmosphere and heat that retains the ocean in its fluid state. To establish these truths let us present in a few words one of the principal late discoveries concerning heat.

Whatever its nature may be, heat dilates bodies. It changes solids into fluids, and fluids into vapour. These changes of form are marked by singular phenomena which we will trace from ice. Let us consider a volume of snow or pounded ice in an open vessel, submitted to the action of great heat. If the temperature of ice is below that of melting ice, it will augment up to zero of temperature. Arrived at this degree the ice will melt successively by new additions of heat. But if care is taken to agitate it till it is all melted, the water will remain at the temperature of zero, the heat communicated by the vessel will not be sensible to the thermometer immersed in it, it will be entirely occupied in rendering the ice fluid. Afterwards the additional heat will raise the temperature of the water and the thermometer till the moment of ebullition. The thermometer will then again become stationary, and all the heat communicated by the vessel will be employed to reduce the water to steam, which will be at the same temperature as the boiling water.

The water produced by the melting of the ice, and the vapours to which the boiling water is changed, absorb at the moment of their formation a great quantity of heat, which re-appears in the return of the aqueous vapours to the state of water, and of water to the state of ice: for steam, when condensing on a cold body, communicates much more heat than would be received from an equal weight of boiling water; and besides it is known that water will preserve its fluid state though its temperature may be many degrees below zero; if then it is a little agitated it transforms itself into ice, and the thermometer plunged into it, mounts to zero from the heat given out during this change.

Without the pressure of the atmosphere melted ice would itself be converted into steam. But this pressure restrains the repulsive force which heat communicates to fluid particles, and retains melted ice under the form of water till the heat is sufficiently great for the repulsive force to overcome the pressure of the atmosphere.

At this instant water boils and becomes steam: the degree of the temperature of boiling water, varies therefore with the pressure of the atmosphere, it is less at the summit of mountains than at the level of the sea, and in a receiver in which the air may be rarified and condensed at pleasure, the heat of boiling water may be increased or diminished at pleasure. Thus heat renders the sea fluid, and the pressure of the atmosphere prevents its being reduced to vapour.

All bodies which can be made to pass from a solid to a fluid state present similar phenomena. But the temperature at which their fusion commences is very different in each of them. Mercury, for example, becomes solid at forty degrees below zero, as is found by experience. It begins to melt at this degree of temperature, it boils at 376, and under the pressure of a column of Mercury of 0.76 metres: so that at this pressure of the atmosphere, the interval of temperature included between fusion and ebullition which is for water 100°, increases to 416° for mercury.

There exist bodies which cannot become fluid by the greatest heat we can induce. There are others which the greatest cold experienced on the earth, cannot reduce to a solid form. Such are the fluids which form our atmosphere, and which, notwithstanding the pressure and the cold to which they have been submitted, have still maintained themselves in the state of vapour. But their analogy with aëriform fluids, to which we reduce a great number of substances by heat, and their condensation by pressure and cold, leaves no doubt but that these atmospheric fluids are extremely volatile bodies, which an intense cold would reduce to a solid state. To make them enter this state, it would be sufficient to remove the Earth from the Sun, as it would be sufficient to bring it nearer, to induce water, and many other bodies, to enter into our atmosphere. These great vicissitudes take place on comets, and principally on those which in their perihelion approach very near the Sun. The nebulosities which surround

them, and the long trains which follow them, are the result of the evaporation of the fluids at their surface. The cold, which is the consequence of this, should temper the excessive heat produced by their proximity to the Sun. And the condensation of the same fluids, when they depart from it, repairs partly the diminution of heat which this separation should produce; so that the double effect of the evaporation and condensation of the fluids, approximates considerably the greatest heat and cold which comets experience at each of their revolutions.

THE

SYSTEM OF THE WORLD.

BOOK II.

On the real Motions of the Heavenly Bodies.

——Provehemur portiu terræque urbesque recedunt.
VIRG. Eneid. Lib. III.

IF man had confined himself to collecting facts alone, science would only have presented a steril nomenclature, and he would never have attained the knowledge of the great laws of nature. It is by comparing phenomena together, and by endeavouring to trace their connection with each other, that he has succeeded in discovering these laws, the existence of which may be perceived even in the most complicated of their effects. Then it is, that nature in discovering herself, has shewn how from a small number of general causes she has

produced the infinite variety of phenomena which have been observed; and thus enabled us to determine those, which successive circumstances will bring to light, and being assured that nothing will interpose between these causes and their effects, we venture to extend our views into futurity, and contemplate the series of events, which time alone can develope. It is only in the theory of the system of the world, that the human understanding has attained to this state of perfection. Let us examine the path that has been followed in this investigation.

CHAP. I.

Of the Motion of Rotation of the Earth.

When we reflect on the diurnal motion to which all the heavenly bodies are subject, we cannot but recognize the existence of one general cause which moves them, or which seems to move them round the axis of the Earth. If we consider that these bodies are insulated, with respect to each other, and placed at very different distances from the Earth, that the Sun and the Stars are at a much greater distance from it than the Moon, and that the variations in the apparent diameters of the planets, indicate great alterations in their distances; and moreover, that the comets traverse the heavens freely in all directions, it will be difficult to conceive that it is the same cause which im-

presses on all these bodies a common motion of rotation. But since the heavenly bodies present the same appearances to us, whether the firmament carries them round the Earth, considered as immovable, or whether the Earth itself revolves in a contrary direction; it seems much more natural to admit this latter motion, and to regard that of the heavens as only apparent.

The Earth is a globe whose radius is only about four thousand English miles; the Sun, as we have seen, is incomparably larger. If its centre coincided with that of the Earth, its volume would embrace the orbit of the Moon, and extend as far again, from which we may judge of its immense magnitude, besides, it distance from us is twenty-three thousand times the semi-diameter of the Earth. Is it not infinitely more simple to attribute to the globe we inhabit, a motion of rotation on its own axis, than to suppose, in a mass so considerable, and so remote, as the Sun, such an extreme rapid motion as would be

requisite to revolve in one day round the Earth? What immense power would it not require to contain it, and counterbalance its centrifugal force? Every one of the stars presents similar difficulties, which are all removed by the rotation of the Earth.

We have seen that the pole of the equator seems to move slowly round that of the ecliptic, from whence results the precession of the equinoxes. If the Earth is immoveable, the pole of the equator is equally so, since it always corresponds to the same point of the terrestrial surface; the ecliptic therefore moves round these poles, and in this motion carries all the heavenly bodies with it. Thus the whole system, composed of so many bodies, differing from each other so much in their magnitude, motions, and distances, would be again subject to a general motion, which disappears, and is reduced to a simple appearance, if we suppose the terrestrial axis to move round the poles of the ecliptic.

Carried on with a velocity which is common to every thing that surrounds us, we are in the case of a spectator placed in a ship that is in motion. He fancies himself at rest, and the shore, the hills, and all the objects placed out of the vessel, appear to him to move. But on comparing the extent of the shore, the plains, and the height of the mountains, with the smallness of his vessel, he recognizes that the apparent motion of these objects, arises from the real motion of himself. The numberless stars which fill the celestial regions, are relatively to the Earth, what the shores and the hills are to the vessel; and the same reasons which convince the navigator of the reality of his own motion, prove to us the motion of the Earth.

These arguments are likewise strengthened by analogy. A rotatory motion has been observed in several planets, and always from west to east, similar to that which the diurnal motion of the heavens seems to indicate in the Earth. Jupiter,

greatly exceeding the Earth in magnitude, moves round its axis in less than twelve hours. An observer on its surface would see the heavens revolve round him in that time; yet that motion would only be apparent. Is it not therefore reasonable to think that it is the same with that which we observe on the Earth? What confirms, in a very striking manner, this analogy, is, that both the Earth and Jupiter, are flattened at the poles. We comprehend, in fact, that the centrifugal force which tends to remove every particle of a body from its axis of rotation, should flatten the Earth at its poles, and elevate it at the equator. This force should likewise diminish that of gravity at the equator; and that this diminution does take place, is proved by experiments which have been made on the lengths of pendulums. Every thing then leads us to conclude, that the Earth has really a motion of rotation, and the diurnal motion of the heavens is merely an illusion which is produced by it;—an illusion similar to that which represents

the heavens as a blue vault, to which all the Stars are fixed, and the Earth as a plane, on which it rests.

Thus astronomy has surmounted the illusions of the senses, and it is not till they have been dissipated by a great number of observations and calculations, that man has at last recognized the motion of the globe which he inhabits, and its true position in the universe.

CHAP. II.

Of the Motion of the Planets round the Sun.

We shall consider at present the phenomena of the motions proper to the planets, and first let us follow the motion of Venus, its apparent diameter, and its phases. In the morning, when it begins to disengage itself from the rays of the Sun, it is seen previous to the rising of this luminary, under the form of a crescent, and its apparent diameter is at its maximum. She is then nearer to us than the Sun, and almost in conjunction with it. Its crescent augments, and its apparent diameter diminishes, in proportion as it recedes from the Sun. Arrived at about * fifty degrees distance from the Sun, it returns towards it, discovering more and more of its enlight-

* 45°.

ened hemisphere to us. Its apparent diameter continues to diminish till the moment when it replunges in the morning into the solar rays.

At this instant Venus appears to us full, and its apparent diameter is a minimum. It is then, in this position, farther from us than the Sun. After disappearing for some time, this planet re-appears in the evening, and produces, in an inverse order, the phenomena it had displayed, previous to its disappearance. Its enlightened hemisphere turns more and more from the Earth, its crescent diminishes, and at the same time its apparent diameter augments, as it becomes more distant from the Sun. When it is about fifty degrees, she returns towards it, its crescent continues to diminish, and its diameter to augment, till it again immerges into the rays of the Sun. Sometimes, during the interval which separates its disappearance in the evening, from its re-appearance in the morning, it is seen under the form of a spot, moving on the Sun's disk. It is

evident, from all these phenomena, that the Sun is nearly in the centre of the orbit of Venus, and that in its motion round the Earth, it carries with it this orbit.

This result, obtained by observations of the phases and apparent diameter of Venus, explains so naturally its alternate, direct, and retrograde motion in longitude, and its whimsical and complicated motion in latitude, that it is impossible to relapse into doubt concerning it. Mercury offers us the same appearances as Venus, therefore the Sun is nearly in the centre of its orbit. These two planets accompany it in its revolution about the Earth, without departing from it above certain limits, which depend on the angles under which their orbits are seen. Those planets which leave the Sun at the greatest possible angular distance, present to us other phenomena. Their diameters are at a maximum in the opposition; they diminish in proportion as they approach the Sun; therefore the Earth is not in the centre of motion of these planets. Pre-

vious to the opposition, this motion, from direct, becomes retrograde; after the opposition, it resumes its direct motion, when the planet, in its approach to the Sun, is about as far distant from it, as at the commencement of its retrogradation; and it is at the moment of opposition, that its retrograde velocity is the greatest. This evidently indicates that the observed motion of these planets, is the result of two motions, alternately in the same and contrary directions, and of which one is governed by that of the Sun. Such are the motions of Mercury and Venus, which, revolving round the Sun, are carried with it round the Earth. It is natural to extend the same law to the other planets, with only this difference, that the Earth, placed without the orbits of Venus and Mercury, is within those of Mars, Jupiter, Saturn, and Uranus. All the appearances of the motions and diameters of these planets, proceed so naturally from this hypothesis, that the mechanism of nature cannot be mistaken in it. That the mo-

tion of the planets round the Sun is almost circular is proved in case of Jupiter, by the eclipses of its satellites.

It has been previously shewn, that these phenomena give the distances of these planets from the Sun, in parts of the mean distance of the Sun from the Earth; and it has been thus found, that these distances vary little in the course of one revolution, and that the motions of these planets are nearly uniform. We are then conducted by the comparison of these phenomena, to place the Sun in the centre of the orbits of all the planets which move round it, while it moves, or appears to move, round the Earth.

CHAP. III.

Of the Motion of the Earth round the Sun.

SHALL we now suppose the Sun accompanied by the planets and satellites in motion round the Earth, or shall we imagine the Earth and other planets to revolve round the Sun? The appearances of the heavenly motions are the same in the two hypotheses, but the second should be preferred, for the following considerations.

The masses of the Sun and of several of the planets, are considerably greater than that of the Earth; it is much more simple to make the latter revolve round the Sun than to put the whole solar system in motion round the Earth. What a complication in the heavenly motions, would the immobility of the Earth suppose! What a rapidity of motion, must be given to Jupiter, to Saturn, which is ten times

farther from the Sun than we are, and to Uranus which is still more remote, to make them every year revolve round us, at the same time they are revolving round the Sun.

This complication and this rapidity of motion disappear by transferring the motion to the Earth: a motion conformable to the general law, by which the small celestial bodies revolve round the large ones which are placed in their vicinity.

The analogy of the Earth with the planets confirms this hypothesis; like Jupiter it revolves on its axis and is accompanied by a satellite. An observer on the surface of Jupiter, would conclude that the solar system was in motion round him, and the magnitude of that planet would render this illusion less improbable than for the Earth. Is it not therefore reasonable to imagine that the motion of the solar system round us, is likewise only an illusion. Let us transport ourselves in imagination to the surface of the Sun, and from thence

let us consider the Earth and the planets. All these bodies will appear to move from west to east; already this identity in the direction indicates a motion of the Earth, but that which demonstrates it evidently is the law which exists between the times of the revolutions of the planets, and their distance from the Sun. They revolve round it slower as their distances are greater, and in such a manner that the squares of the periodic time are in proportion to the cubes of their mean distances. According to this remarkable law, the length of a revolution of the Earth, supposing it in motion round the Sun, should be exactly a sidereal year. Is not this an incontestable proof that the Earth moves like the other planets and is subject to the same laws? For would it not be very strange to suppose the terrestrial globe which hardly subtends a visible angle at the Sun, immovable amidst the other planets which are revolving round it, and that the Sun should be carried with them about the Earth?

Ought not the force which balances the centrifugal force and retains the planets in their respective orbits, likewise to act upon the Earth ? and must not the Earth oppose to this action the same centrifugal force ? Thus the consideration of the celestial motions as observed from the Sun, leaves no doubt of the real motion of the Earth. An observer on the surface of the Earth has another evident proof of its motion in the phenomenon of the aberration, which is a necessary consequence of it, as we shall now explain.—About the end of the seventeenth century, Roemer observed that the eclipses of the satellites of Jupiter happened sooner about the oppositions of this planet, and later towards the conjunctions ; this led him to suspect that light was not transmitted instantaneously from those bodies to the Earth, but that it employed a perceptible interval of time to traverse the diameter of the orbit of the Sun. In fact, Jupiter being in his oppositions nearer to us than in the conjunctions, by a quantity equal to this

diameter, the eclipses ought to happen sooner to us in the first case than in the latter, by the time which the light takes to traverse this orbit. The law of retardation observed in these eclipses, answers so exactly to this hypothesis, that it is impossible to refuse assent to it. It appears that light employs * 571″ in coming from the Sun to the Earth.

Now an observer at rest would see the stars according to the directions of their rays, but this will not be the case on the supposition that he partakes of the motion of the Earth. To reduce this case to that of the observer at rest, it is sufficient to assign in a contrary direction, both to the stars, to the light, and to the observer himself, a motion equal to that by which he is impelled, which would not change the apparent position of the stars; for it is a general law of optics that if all the bodies of a system are impelled by a common motion, there will result no

* 8′ 5″.

change in their respective situations. Let us imagine, then, that a motion equal and contrary to that of the observer, be given to the rays of light, and generally to all the other bodies, and let us see what phenomena should result in the apparent position of the stars. We may leave out of the question the diurnal motion of the Earth, which is not even at the equator $\frac{1}{230}$th part of that in its orbit round the Sun. We may here suppose also without sensible error that all the rays which each point of the disk of a heavenly body transmits to us are parallel to each other, and to that ray which would come from the centre of the star to the centre of the Earth if it were transparent. Thus the phenomena which the stars would present to an observer placed at the centre of the Earth, and which depend on the motion of light combined with that of the Earth, are nearly the same for every observer on its surface. Moreover we may neglect the small excentricity of the terrestrial orbit.

In the interval of * 571″ that light employs to traverse the terrestrial orbit, the Earth describes a small arc of this orbit equal to † 62″ 5; now it follows from the law of the composition of motion, that if through the centre of a star we imagine a small circle parallel to the ecliptic, and whose diameter subtends in the heavens an angle of ‡ 125″, the direction of the motion of light combined with the motion of the Earth, and applied in a contrary direction, will meet this circumference in a point where it is intersected by a plane drawn through the centre of the star tangentially to the terrestrial orbit. The star therefore should move upon this circumference, and describe it every year in such a manner that it should constantly be less advanced by § 100° than the Sun in its apparent orbit.

This phenomenon is exactly that which we have explained in Chap. XI. Book I. from the observations of Bradley, to whom

---

* 8′ 5″. † 20″ 2. ‡ 40″ 5. § 90°.

we are indebted both for the discovery and cause. To reduce these stars to their true position, it is sufficient to place them in the centre of the small circle they appear to us to describe.

Their annual motion, therefore, is only an illusion produced by the combination of the motion of light with that of the Earth. The relation of this motion with the position of the Sun, would lead to a suspicion that it was only apparent, but the foregoing explanation proves it beyond a doubt. It affords a sensible demonstration of the motion of the Earth round the Sun, in the same manner as the increase of degrees and of the force of gravity in going from the equator to the poles, proves its revolution on its axis.

The aberration of light affects the positions of the Sun, the planets and their satellites, and comets, but in a different manner according to their particular motions. To divest them of this, and to obtain the true position of the stars, let us suppose at every instant, a motion impress-

ed on all these bodies equal and contrary to that of the Earth, which thus may be supposed at rest, this, as observed above, neither alters their respective positions nor appearances. Then it is evident that a heavenly body, at the moment we observe it, is no longer in the direction of the luminous ray which strikes our sight; it has left it in consequence of its real motion combined with that of the Earth which we supposed impressed in a contrary direction. The combination of these two motions, as seen from the Earth, forms the apparent or, as it is termed, the geocentric motion. We shall have then the true position of the object, by adding to the observed geocentric longitude and latitude, its geocentric motion in longitude and latitude, for the interval of time which light employs to come from the heavenly body to the Earth. Thus the centre of the Sun seems constantly less advanced by *62″ 5, in its orbit, than if its

*20″.2.

light was transmitted to us instantaneously.

The aberration of light changes the relations of celestial phenomena, both as to space and as to duration. At the moment we see them they no longer exist. We do not see the termination of the eclipses of Jupiter's satellites till *25′ or 30′ after they have recovered their light; and the variations of light of some of the changeable stars precede by many years the instant of their observations. But the cause of these illusions being well understood, we can always refer the phenomena of the solar system to their true place and epoch.

The consideration of the celestial motions leads us, then, to displace the Earth from the centre of the world where we had placed it, deceived by appearances, and by the natural propensity of man to regard himself as the principal object in nature. The globe which we inhabit is a planet in motion on itself and round the Sun.

---

* 36′ or 42″.

In considering it in this point of view, all the phenomena are explained in the most simple manner, all the celestial motions become uniform, and the analogies are preserved. Like Jupiter, Saturn, and Uranus, the Earth is accompanied by a satellite, it turns on itself like Venus, Mars, Jupiter, and Saturn, and probably the other planets; it like them borrows its light from the Sun, and moves round it in the same direction, and according to the same laws. Finally the hypothesis of the Earth's motion unites in its favour simplicity, analogy, and every thing which characterizes the true system of nature. We shall see that following it in its consequences, the celestial phenomena are brought, even in their minutest details, to one single law, of which they are only the necessary developments. The motion of the Earth will thus acquire all the certainty of which physical truths are susceptible. And it may result either from the great number and variety of phenomena which it explains, or from the sim-

plicity of the laws on which it is made to depend. None of the branches of natural knowledge unite these advantages in a higher degree than the theory of the system of the world founded on the motion of the Earth. This motion ennobles our conceptions of the universe, by affording for a measure of the distances of the heavenly bodies, an immense base—the diameter of the terrestrial orbit. By this we have accurately determined the dimensions of the planetary orbs.

Thus the motion of the Earth, after having, by illusions of which it is itself the cause, retarded our knowledge of the planetary motion for a great length of time, at last conducted us to them, and that in a more accurate manner than if we had been placed in the centre of their system.

Nevertheless the annual parallax of the stars, or the angle which the diameter of the terrestrial orbit would subtend at this centre, is insensible, and does not

amount to *6″ even relatively to those stars which from their extreme brilliance appear to be nearest to us. They are, therefore at least a hundred thousand times farther from us than the Sun. Their prodigious brightness at so immense a distance, proves to us that they do not, like the planets and their satellites, borrow their light from the Sun, but that they shine with their own light; so that they are so many suns scattered in the immensity of space, and which, similarly to ours, may be the foci of so many planetary systems. It would in fact be sufficient to place ourselves upon the nearest of these stars in order to see the Sun only as a luminous object, the diameter of which would be less than the thirtieth of a second.

It results from the immense distance of the stars, that their motion in right ascension and declination, are only appearances produced by the motion of the Earth's

* 1″ 9.

axis of rotation. But some stars appear to have motions proper to themselves, and it is probable that they are all in motion as well as the Sun which carries with it the whole system of planets in space, in the same manner as each planet draws its satellites in its motion round the Sun.

CHAP. IV.

Of the Phenomena which arise from the Motion of the Earth.

From the point of view in which the comparison of the celestial observations has placed us, let us consider the heavenly bodies, and show the perfect identity of their appearances with those which we observe. Whether the heavens revolve round the axis of the world, or the Earth moves itself in a contrary direction to the heavens, supposed at rest, it is clear that the stars will present themselves to us in the same manner. There will be no other difference but that in the first case they will come and place themselves successively over the different terrestrial meridians, which in the second will place

themselves under the stars. The motion of the Earth being common to all bodies situated on its surface, and to the fluids which cover it, their relative motions are the same as if the Earth was at rest. Thus in a vessel whose motion is uniform, every thing moves as if the vessel were at rest. A projectile thrown directly upwards falls on the same spot from which it was projected, it seems to describe a vertical line to those in the vessel, but seen from the shore it really describes a parabolic curve. Thus the rotation of the Earth cannot be sensible on its surface, except by the effects of the centrifugal force which flattens the terrestrial spheroid at the poles, and diminishes the force of gravity at the equator; two phenomena with which the measure of the degrees at the meridian, and of the pendulum, have made us acquainted.

In the revolution of the Earth round the Sun, its centre and all the points of its axis of rotation, being moved with velocities equal and parallel, this axis re-

mains always parallel to itself: in impressing then at every instant to all the parts of the Earth a motion equal and contrary to that of its centre, it would rest immovable like its axis of rotation; this impressed motion does not change at all the appearances of that of the Sun, it only transports in a contrary direction to the Sun, the real motion of the Earth. The appearances are consequently the same in the hypothesis of the Earth at rest, and in that of its motion round the Sun. To follow more particularly these appearances, let us imagine a radius drawn from the centre of the Sun to that of the Earth: this radius will be perpendicular to the plane which separates the hemisphere which is enlightened from that which is in obscurity. A spectator at the point where this intersects the terrestrial surface, will see the Sun perpendicularly above him, and every point of the terrestrial parallel through which this radius successively passes in consequence of its diurnal motion, will have at noon the Sun in its zenith. Thus

whether the Sun turns round the Earth, or the Earth round the Sun and on its own axis, this axis preserving a parallel position, it is evident that this radius will trace the same curve on the surface of the Earth, it will in each case cut the same parallels to the equator when the Sun has the same apparent longitude: this luminary will be equally elevated above the horizon, and the days will be of the same length. Thus the seasons and the days are the same in the hypothesis of the immobility of the Sun, or of its motion round the Earth: and the explanation of the seasons will be equally intelligible by either hypothesis.

The planets all move in the same direction round the Sun, but with different velocities, but the length of their revolutions increase in a greater ratio than their distances from the Sun. Jupiter, for instance, employs nearly twelve years to perform its revolution, but the radius of his orbit is only five times greater than that of the Earth; its real velocity is therefore less

than that of the Earth; this diminution of velocity in the planets as they recede from the Sun applies generally to all of them, from Mercury, which is the nearest, to Uranus, which is the most remote; and it results from the laws which we shall hereafter demonstrate, that the mean velocities of the planets are reciprocally as the square roots of their mean distances from the Sun.

Let us consider a planet whose orbit is surrounded by that of the Earth, and follow it from its superior to its inferior conjunction; its apparent or geocentric motion is the result of its real motion combined with that of the Earth considered as moving in a contrary direction. In the superior conjunction, the real motion of the planet is contrary to that of the Earth, its geocentric motion is then the sum of the two motions, and it has then the same direction as the geocentric motion of the Sun, which results from the real motion of the Earth transferred in a contrary direction to the Sun, and thus

the apparent motion of the planet is direct.

In the inferior conjunction the motion of the planet has the same direction as that of the Earth; and as it is greater, the geocentric motion preserves the same direction, but it is only the excess of the real motion of the planet above that of the Earth, it has therefore a motion contrary to that of the Sun, and consequently it is retrograde.

It is easy to conceive that in the interval from the direct to the retrograde motion, the planet should appear without motion, or stationary; and that this will happen between the greatest elongation and the inferior conjunction, when the geocentric motion of the planet, resulting from its real motion and that of the Earth applied in a contrary direction, is directed in the same line as the visual ray to the planet. These phenomena are entirely conformable to the motions that are observed to take place in the planets Mercury and Venus.

The motion of the planets whose orbits comprehend that of the Earth, has the same direction in their oppositions as the motion of the Earth, but it is less, and being combined with this last motion applied in a contrary direction, it takes a motion contrary to its primitive direction, the geocentric motion of the planet is then retrograde, in the conjunctions it is direct, the same as in the superior conjunctions of Mercury and Venus.

In transferring to the stars (but in a contrary direction) the motion of the Earth, they should describe every year a circumference equal and parallel to the terrestrial orbit, and of which the diameter should subtend an angle equal to that which this orbit subtends at the distance of the star. This apparent motion has a great resemblance to that which results from the combination of that of the Earth with that of light, and by which the stars seem annually to describe a circle parallelto the ecliptic, the diameter of which subtends

an angle of *125″, but it differs in this, that these stars have the same position as the Sun on the first circumference, whereas in the second they are less advanced than the Sun by †100°. By this circumstance the two motions may be distinguished from each other, and it appears that the first is insensible. The immense distance of the stars from us rendering insensible the angle which the terrestrial orb subtends when viewed from them.

The axis of the world being nothing more than the prolongation of the Earth's axis of rotation, we should refer to this axis the motion of the poles of the celestial equator, indicated by the phenomena of *precession* and *nutation*, which see in Chap. XI. of the first Book. Thus at the same time that the Earth moves on its own axis, and round the Sun, its axis of rotation moves very slowly round the poles of the ecliptic: but subject to small

---

* 40″ 5. † 90°

oscillations of which the period is the same as that of the motion of the nodes of the lunar orbit. But this motion is not peculiar to the Earth alone, for we have seen in the fourth Chapter of the first Book, that the axis of the Moon moves in the same period round the poles of the ecliptic.

CHAP. V.

Of the Figure of the planetary Orbits, and of the Law of their Motions round the Sun.

NOTHING would be more easy than to calculate the position of the planets at any given moment, from the preceding data, if their motions round the Sun were circular and uniform. But they are subject to very perceptible inequalities, the laws of which form one of the most important objects of astronomy, and are the only clew that can conduct us to the general principles of the celestial motions.

To recognize these laws in those appearances which the planets offer to us, it is requisite to deprive their movements of the effects of the Earth's motion, and to refer to the Sun their position observed from different points of the terrestrial orbit. It is then necessary, first of all to determine the dimensions of this orbit and the law of the Earth's motion.

It has been shewn in the second Chapter of the first Book, that the apparent orbit of the Sun is an ellipse, of which the centre of the earth occupies one of the foci; but as the Sun is really immovable it must be placed in the focus of the ellipse, and the Earth at the circumference. The apparent motion of the Sun would be the same, and to obtain the position of the Earth, as seen from the centre of the Sun, it would be sufficient to augment by two right angles the position of that body. It has been also seen that the Sun seems to move in its orbit in such a manner that the radius vector which joins its centre to that of the Earth, appears to trace round it areas proportionate to the times in which they are described: but in reality these areas are traced round the Sun. In general, all that has been said in the chapter already cited, upon the excentricity of the solar orbit and its variations on the position and motion of its perigee, ought to be applied to the terrestrial orbit, observing only that the Earth's perigee is at the dis-

tance of two right angles from that of the Sun. The figure of the terrestrial orbit being thus known, let us see how the knowledge of the orbits of the other planets has been arrived at. Let us take for example, the planet Mars, which by the great excentricity of its orbit, and its proximity to the Earth, is very well calculated to discover to us the laws of the planetary motions.

The motion of Mars round the Sun and its orbit would be known, if we could obtain for any instant the angle which its radius vector makes with an invariable straight line passing through the Sun's centre, and the length of this radius. To simplify this problem, we choose those positions of Mars in which one of these quantities can be found separately; and it is this which takes place very nearly in the oppositions in which this planet is seen opposite the same points in the ecliptic, to which it might be referred from the centre of the Sun. The difference of the motion of Mars and the Earth makes the planet correspond to different points of the

heavens, in its successive oppositions; by comparing then a great number of observed oppositions, the law which exists between the time and the angular motions of Mars round the Sun may be discovered; this motion is called heliocentric. Analysis offers for this purpose several methods, which become simple in the present case from the consideration that as the principal inequalities of Mars become the same at each of its sidereal revolutions, their sum may be expressed by a rapidly-converging series of the sines of the angles multiplied by its mean motion, the co-efficients of which series it is easy to obtain, by means of some select observation. The law of the radius vector of Mars may be obtained by comparing the observations of this planet towards the quadratures, or when, being about * 100° of the Sun, this radius subtends the greatest angle. In the triangle formed by the right lines which join the centres of the Earth, the Sun and Mars, the angle at the

* 90°.

law of the heliocentric motion of Mars gives the angle at the Sun, and the radius vector of Mars may be concluded in parts of that of the Earth, which is itself given in parts of the mean distance of the Earth from the Sun. The comparison of a great number of the radii vectores thus determined, will give the law of their variations, corresponding to angles which they form with an invariable right line, and the figure of the orbit may be traced.

It was by a similar method that Kepler discovered the elongation and excentricity of the orbit of Mars: he conceived the fortunate idea of comparing its figure with that of the ellipse, placing the Sun in one of the foci, and the numerous observations of Tycho, exactly represented in the hypothesis of an elliptic orbit, left no doubt concerning the truth of this hypothesis.

That extremity of the greater axis which is nearest to the Sun is called the perihelion, and that which is farthest the aphelion. Earth is given directly by observation, the

It is in the perihelion that the angular velocity of Mars round the Sun is the greatest; it diminishes as the radius vector augments, and is the least at the aphelion: comparing this velocity with the powers of the radius vector, Kepler found that it is proportionate to its square: so that, the product of the daily heliocentric motion of Mars by the square of its radius vector is always the same.

This product is the double of the small sector which this radius describes every day about the Sun: the area then which it describes, departing from an invariable line passing through the centre of the Sun, increases as the number of days passed since the epoch when the planet was upon this line. This is what Kepler announced, in establishing that the areas described by the radius vector of Mars are proportional to the time.

These laws of the motion of Mars are the same with those of the apparent motion of the Sun, which have been developed in the second Chapter of the first Book; they therefore apply equally to the Earth. It

was natural to extend them to the other planets.

Kepler then established as the fundamental laws of the motions of these bodies, the two following ones, which all observations have confirmed.

The planetary orbits are ellipses, of which the centre of the Sun occupies one of the foci.

The areas described round this centre by the radii vectores of the planets are proportionate to the time employed to describe them.

These laws are sufficient to determine the motions of the planets round the Sun. But it is necessary to know for each of them, seven quantities, which are called *elements of the elliptic motion.* Five of these elements, relative to the motion of the ellipse, are, first, the duration of the sidereal revolution; second, half the greater axis of the orbit, or the mean distance of the planet from the Sun; third, the excentricity from which the greatest equation of the centre is derived; fourth, the mean

longitude of the planet at any given epoch; fifth, the longitude of the perihelion at the same epoch. The two other elements relate to the position of the orbit, and are, first, the longitude at a given epoch, the nodes of the orbit, or the points of intersection with a plane, which is usually supposed to be that of the ecliptic; second, the inclination of the orbit to this plane. There are then forty-nine elements to determine for the entire system of the known planets. The following table presents all these elements for the beginning of 1750.

The inspection of this table shews us, that the duration of the planetary revolutions, increase with their mean distances from the Sun: this made Kepler suspect that they were related to these distances by some proportion, which he proposed to discover. After a great number of trials continued through seventeen years, he at last found that *the squares of the times of the planetary revolutions, are to each other as the cubes of the greater axes of their orbits.*

Such are the laws of the motion of the planets; fundamental laws which have given a new aspect to astronomy, and have led to the discovery of universal gravitation.

The planetary ellipses are not unalterable. Their greater axes appear to be always the same; but their excentricities, their inclinations to a fixed plane, the position of their nodes and of their perihelia are subject to variations, which appear up to the present time to have increased proportional to the times. These variations only becoming sensible through the lapse of ages, have been called *secular inequalities*. There is no doubt of their existence, but modern observations not being sufficiently distant from each other, and ancient observations not being sufficiently exact, to ascertain them with precision, there remains yet a great incertitude regarding their quantity. The following tables offer values which appear to satisfy most correctly the mass of these observations.

Periodical inequalities have been like-

wise remarked, which disturb the elliptic motions of the planets: that of the Sun is a little altered, as has been seen in the preceding Book, but these inequalities are principally sensible in the two largest planets, Jupiter and Saturn. Comparing modern observations with the ancient, astronomers have remarked a diminution in the duration of Jupiter's revolution, and an increase in that of Saturn. Modern observations compared with each other give a contrary result, which seems to indicate great inequalities in the motions of these planets, the periods of which are very long. Even in this century the duration of Saturn's revolution has appeared different, following the points of the orbit from which the departure of the planet has been fixed; its returns to the vernal equinox have been more rapid than to the autumnal. Finally, Jupiter and Saturn experience inequalities which amount to many minutes, and which appear to depend on the situation of these planets, whether among themselves or

with relation to their perihelia. Thus every thing announces that in the planetary system, independent of the principal cause which makes the planets move round the Sun in elliptic orbits, there exist particular causes which disturb their movements, and which alter at length the elements of their ellipses.

TABLE

OF THE ELLIPTIC MOTIONS OF THE PLANETS.

Duration of their sidereal revolution.

	Days.	yrs.	days.	hrs.	min.	sec.	
Mercury .	87.969255	0	87	23	15	44	0
Venus .	224.700817	0	224	16	49	11	0
The Earth	365.256384	1	0	6	9	8	0
Mars .	686.979579	1	321	23	30	35	6
Jupiter .	4332.602208	11	317	14	27	10	7
Saturn .	10759.077213	29	174	1	51	11	2
Uranus .	30689.000000	84	29	0	0	0	0

Semi major axes of their orbits, or their mean distances.

Mercury . .	0.387100
Venus . .	0.723332
The Earth . .	1.000000
Mars . .	1.523693
Jupiter . .	5.202592
Saturn . .	9.540724
Uranus . .	19.183620

Proportion of the excentricity of the

semi major axes, for the beginning of the year 1750.

Mercury . .	0.205513
Venus . .	0.006885
The Earth . .	0.016814
Mars . .	0.093808
Jupiter . .	0.048877
Saturn . .	0.056223
Uranus . .	0.046683

The secular variations of this proportion. (The sign — indicates a diminution.)

Mercury . .	0.000003369
Venus . .	—0.000062905
The Earth . .	—0.000045572
Mars . .	0.000090685
Jupiter . .	0.000134245
Saturn . .	—0.000261553
Uranus . .	—0.000026228

The mean longitudes at the commencement of 1750. These longitudes are reckoned from the mean vernal equinox at the epoch of the 31st December, 1749, at noon, mean time at Paris.

	deg.	deg.	min.	sec.	
Mercury .	281.3194	253	11	24	8
Venus .	31.4963	46	20	48	0

	deg.	deg.	min.	sec.	
The Earth .	311.1218	280	0	34	5
Mars .	24.4219	21	58	49	9
Jupiter .	4.1201	3	42	29	0
Saturn .	257.0438	231	20	21	9
Uranus .	353.9610	228	33	53	6

Longitudes of the perihelion at the beginning of 1750.

	deg.	deg.	min.	sec.	
Mercury .	81.7401	73	33	57	9
Venus .	141.9759	127	46	41	9
The Earth .	309.5790	278	37	15	9
Mars .	368.3005	331	28	13	6
Jupiter .	11.5012	10	21	3	8
Saturn .	97.9466	88	9	6	9
Uranus .	185.1262	166	36	48	8

The sidereal and secular motion of the perihelion, (the sign — indicates a retrograde motion.)

	sec.	min.	sec.	
Mercury .	1735.50	9	22	3
Venus .	—699.07	3	46	4
The Earth .	3671.63	19	49	6
Mars .	4834.57	26	6	4
Jupiter .	2030.25	10	57	8
Saturn .	4967.64	26	49	5
Uranus .	759.85	4	6	1

The inclination of the orbit to the ecliptic at the beginning of 1750.

	deg.	deg.	min.	sec.
Mercury .	7.7778.	7	0	0
Venus .	3.7701	3	23	35
The Earth .	0.0000	0	0	0
Mars .	2.0556	1	51	0
Jupiter .	1.4636	1	19	2
Saturn .	2.7762	2	29	54
Uranus .	0.8599	0	46	26

The secular variation of the inclination to the true ecliptic.

	sec.	sec.	
Mercury . .	55 09	17	50
Venus . .	13.80	4	47
The Earth . .	0.00	0	0
Mars . .	—4.45	—1	4
Jupiter . .	—67.40	—21	8
Saturn . . .	—47.87	—15	5
Uranus . .	9.38	3	0

Longitude of the ascending node upon the ecliptic at the beginning of 1750.

	deg.	deg.	min.	sec.	
Mercury .	50.3836	45	20	42	8
Venus .	12.7093	74	26	18	0

	deg.	deg.	min.	sec.	
The Earth .	0.0000	0	0	0	0
Mars .	52.9577	47	38	38	0
Jupiter .	108.8062	97	55	32	0
Saturn .	123.9327	111	32	21	9
Uranus .	80.7015	72	37	52	8

The sidereal and secular motion of the node upon the true ecliptic.

	sec.	min.	sec.	
Mercury .	—2332.90	12	35	8
Venus .	—5673.60	30	38	2
The Earth .	0.00	0	0	0
Mars .	—7027.41	37	58	0
Jupiter .	—4509.50	24	21	0
Saturn .	—5781.54	31	13	2
Uranus .	—10608.00	57	16	9

CHAP. VI.

Of the Figures of the Orbits of Comets, and of the Laws of their Motion round the Sun.

The Sun being the focus of the planetary orbits, it is natural to suppose it equally that of the orbits of comets. But these celestial bodies disappear after having been visible some months; their orbits instead of being nearly circular, like those of the planets, are very excentric, and the Sun is extremely near to that part in which they are visible. The ellipse, by the infinite varieties which it admits of from the circle to the parabola may suit these different orbits. Analogy leads us then to suppose that comets move in elliptic orbits of which the Sun occupies one of the foci, and to consider them as following the same laws with the planets, so that the areas described by their radii vectores are equal in equal times.

It is almost impossible to know the duration of the revolution of a comet, and consequently the greater axis of its orbit, by the observation of only one of its appearances; the area which its radius vector describes in a given time cannot then be determined rigorously. But it should be considered that the small portion of the ellipse described by the comet during its appearance may be supposed to coincide with a parabola, and thus its motion during a short interval may be calculated as if it was parabolical.

According to the laws of Kepler, the sectors described in the same time by two planets, are to each other as the areas of their ellipses divided by the squares of the times of the revolution, and these squares are as the cubes of their semi-major axes. It is easy to conclude that if we imagine a planet moving in a circular orbit, of which the radius is equal to the perihelion distance of a comet, the sector described by the radius vector of the comet will be to the corresponding sector described by the

radius vector of the planet, as the square root of the aphelion distance of the comet is to the square root of the semi-major axis of its orbit, a relation which, when the ellipse changes to a parabola, becomes that of the square root of two to unity.

The relation of the sector of the comet to that of the imaginary planet is thus obtained, and it is easy by what has been already said, to get the proportion of this last sector, to that which the radius vector of the Earth describes in the same time. The area described by the radius vector of the comet may then be determined for any instant whatever, setting out from the moment of its passage through the perihelion, and its position may be fixed in the parabola which it is supposed to describe.

Nothing more is necessary but to deduce from observation the elements of the parabolic motion, that is to say, the perihelion distance of the comet, the position of the perihelion, the instant of its passage through the perihelion, the inclination of

its orbit to the ecliptic, and the position of its nodes. The investigation of these five elements presents much greater difficulties than that of the elements of the planets, which being always visible, and having been observed during a long succession of years may be compared when in the most favourable position for determining these elements, instead of which comets only appear for a very short time, and frequently in circumstances where their apparent motion is rendered very complicated, by the real motion of the Earth, which always carries us in a contrary direction.

Notwithstanding all these difficulties, it is possible to determine the elements of the orbits of comets by different methods. Three complete observations are more than sufficient for this object; others only serve to confirm the accuracy of these elements, and the truth of the theory which we have just explained. Above four and twenty comets, the numerous observations of which are exactly

represented by this theory, have confirmed it beyond all doubt.

It appears therefore that comets which have been considered as meteors for many years, are of the same nature as planets; their motions and their returns are regulated by the same laws as planetary motions.

Let us observe here how the true system of nature, as it is developed, becomes more and more confirmed.

The simplicity of the celestial phenomena on the supposition of the Earth's motion, compared to their extreme complication, on that of its being stationary, renders the first of these suppositions extremely probable. The laws of elliptic motion then common to the planets and the Earth, augment this probability, which becomes yet greater from the motion of comets, subjected by this hypothesis to the same laws.

Comets do not always move in the same direction like the planets. The real motion of some is direct, of others

retrograde. The inclination of their orbits is not confined within a narrow zone, like that of the planetary orbits; they present every variety of inclination, from an orbit coincident with the plane of the ecliptic, to that perpendicular to it.

A comet is recognized when it reappears by the identity of the elements of its orbit with those of the orbit of a comet already observed. If its perihelion distance, the position of its perihelion, its nodes, and the inclination of its orbit are very nearly the same, it is probable that the comet which appears is that which had been observed before, and which, having receded to such a distance as to be invisible, returns to that part of its orbit nearest to the Sun. The duration of the revolution of comets being very long, and having been observed with very little care, till within about two centuries; the period of the revolution of one comet only is known with certainty, that of 1682, which had been already observed in 1607 and 1531, and which has reappeared in 1759. This comet takes

about 76 years to return to its perihelion; therefore taking the mean distance of the Sun from the Earth as unity, the greater axis of its orbit is 35.9, and as its perihelion distance is only 0.58, it recedes from the Sun at least thirty-five times more than the Earth, describing a very excentric ellipse. Its return to the perihelion has been longer by thirteen months from 1531 to 1607, than from 1607 to 1682; it has been eighteen months shorter from 1607 to 1682, than from 1682 to 1759. It appears then that causes similar to those which alter the elliptic motion of the planets, disturb that of comets in a yet more perceptible manner.

The return of some other comets has been suspected: the most probable of these returns was that of the comet of 1532, which has been believed to be the same with that of 1661, and the revolution of which was fixed at 129 years; but this comet not having reappeared in 1790, as was expected, there is

great reason to believe that these two comets were not the same. This ought to render us very circumspect in pronouncing on the identity of two observed comets. Let us try to calculate the probability of this identity when the elements are a little different.

Suppose the difference to be only a degree in the inclination of the orbit, and in the places of the ascending node and perihelion, and a hundredth of a degree in the perihelion distance, the mean distance of the Sun and Earth being taken as unity: suppose again, that the errors of the elements deduced from observation, and the alteration which these elements might experience in the interval of two appearances of the comet be included within these limits, so that nothing should hinder us from considering the two comets as the same.

The inclination of the orbit of a new comet to the ecliptic may vary from zero to the semi circumference but above one

hundred* degrees of inclination the motion changes its direction; thus, whether the motion be direct or retrograde, may be indicated by the inclination alone; the probability that the inclination of the orbit of a new comet should not separate more than a degree from the orbit of a preceding one, is then equal to $\frac{1}{100}$. The position of the ascending node of a comet may vary from zero to † 400°, the probability then that it should not differ more than a degree from that of the node of a comet observed before, is therefore $\frac{1}{200}$°.

In a similar manner the probability that the position of the perihelion of a new comet should not differ more than a degree from that of a former one, is $\frac{1}{100}$. We shall suppose that the perihelion distance may equally vary in the interval comprised between zero and 1.5. Comets have in fact been seen, the perihelion distance of

---

* 90°. † 360°.

which has surpassed 1.5 ; but these cases are sufficiently rare for us to dispense with any attention to them in this trial of calculation, as a greater perihelion distance almost always renders the comet invisible. The probability that the perihelion distance of a new comet should not differ more than one hundredth of a degree from the perihelion distance of a comet formerly observed, will then be nearly $\frac{4}{300}$. Thus the probability that the elements of a new comet should not differ from those of a preceding comet beyond these limits, will be the product of the four numbers, $\frac{1}{100}$, $\frac{1}{200}$, $\frac{1}{200}$, $\frac{1}{300}$, and consequently will be equal to a fraction, the numerator of which being unity, the denominator is equal three hundred millions.

The theory of chances gives the following rule to know the probability of a new comet being the same with one observed before: multiply this fraction by the number of comets visible, and not yet observed, augmented by unity; divide unity by this product *plus* one, the quotient will be the probability sought.

If the limits of the errors of the elements deduced from observation are greater than the preceding, instead of the fraction one divided by three hundred millions, the product of this fraction by that of four numbers, which express how often each limit contains the preceding supposed limit, must be employed.

The number of comets visible and not yet observed being unknown, it is impossible to calculate the probability we are now considering, nevertheless we may believe that they do not exceed a million. Supposing it equal to this quantity, there are 300 to one, that a comet, the elements of which do not differ from those of one formerly observed more than the preceding quantities, is the same. Comparing thus the elements of the comets of 1607 and 1682, Halley was able to announce, with a probability equal to $\frac{1200}{1201}$, that they were the same, and that it would reappear towards the middle of this century. The fear of being deceived, though very

small, almost vanished when he recognized nearly the elements of this comet in that observed in 1531, and it has totally disappeared to us who have seen the comet in 1759.

But it is not thus with the comet of 1532; its elements have been determined upon the observation of Appian and Frucastor, and their observations are so inaccurate that they leave an uncertainty of *41° upon the situation of the nodes, of †10° upon its inclination, of ‡32° on the position of its perihelion, and of §0°.225 upon its perihelion distance. It is necessary therefore to multiply the fraction unity divided by three hundred millions, by the product 41.10.22.17, which reduces it to 0.000517; supposing then that there are yet a thousand comets visible and not yet observed, which is not improbable, the probability that the two comets of 1532 and 1661 are the same would be $\frac{1}{4}$, a probability much too small to pronounce their identity. Therefore we ought not

---

* 36° 54′. † 9°. ‡ 19° 48′. § 15′.

to be surprised that this comet has not reappeared in these last years.

The nebulosity with which these comets are almost always surrounded, seems to be formed by the vapours which the solar heat raises on their surface. It is imagined that the great heat which they experience towards their perihelion, rarifies the particles which have been congealed by the excessive cold of the aphelion.

It appears also that the trains of comets are only these vapours elevated to a considerable height by this rarefaction, combined, either with the solar rays, or with the dissolution of those vapours in the fluid which reflects the zodiacal light to us. This seems to result from the direction of their trains, which are always beyond the comet, relatively to the Sun, and which only becoming visible near their perihelion, are not at a maximum till after their passage through this point, when the heat communicated to the comet by the Sun, is encreased by its duration, and by the proximity to this luminary.

CHAP. VII.

Of the Laws of the Motion of Satellites round their Planets.

We have explained in the Sixth Chapter of the preceding Book, the laws of the motion of the Earth's satellite and of its principal irregularities. It now remains to consider those of the motion of the satellites of Jupiter and Saturn.

If the semidiameter of the equator of Jupiter, supposed *60″.185 at the mean distance of the planet from the Sun, be taken as unity, the mean distance of its satellites from its centre will be nearly:

I. Satellite . .	5.697300
II. Satellite . .	9.065898
III. Satellite . .	14.461628
IV. Satellite . .	25.436000

* 19.″5

The duration of their siderial revolutions are :

	J.	Days.	hrs.	min.	sec.
I. Satellite	1.769137787069931	1	18	27	30
II. Satellite	3.551181016734509	3	13	13	43
III. Satellite	7.154552807541524	7	3	42	28
IV. Satellite	16.689019396008634	16	16	32	9

The synodical revolutions of these satellites, or the intervals of the return of their mean conjunctions with Jupiter, may be easily concluded from the duration of their sidereal revolutions, and that of the sidereal revolution of Jupiter.

At the beginning of 1700, the mean longitudes of the satellites were :

	Deg.	Deg.	min.	sec.
I. Satellite .	85.8491	77	15	51
II. Satellite .	83.5827	75	18	17.9
III. Satellite .	182.4495	161	12	16.3
IV. Satellite .	253.1545	23	50	20.5

Comparing the distances of the four satellites of Jupiter, with the duration of their revolution, the same beautiful proportion has been observed between these

quantities, that exists between the mean distances of the planets from the Sun, and the duration of their revolutions, that is to say, *the squares of the times of the sidereal revolutions of the satellites are to each other, as the cubes of their mean distances from Jupiter.*

The frequent eclipses of the satellites have furnished astronomers with the means of ascertaining their motions with a precision that could not have been expected from the observation of their distance from Jupiter. They have produced the following results:

The ellipticity of the orbit of the first satellite is insensible, its plane coincides nearly with that of the equator of Jupiter, the inclination of which to the orbit of the planet is *4.°4444.

The ellipticity of the orbit of the second satellite is equally insensible. Its inclination to the orbit of Jupiter varies as well

---

* 3° 33′ 20″.

as the position of its nodes. All these variations are represented nearly by supposing the orbit of the satellite inclined about * 5182″ to the equator of Jupiter, and giving to its nodes a retrograde motion on this plane, the period of which is about thirty Julian years.

A slight ellipticity is observable in the orbit of the third satellite. The extremity of its greater axis nearest to Jupiter, and which is called perijove, has a direct motion, and the excentricity of the orbit appears subject to very perceptible alterations. Towards the end of the last century, the equation of the centre was at its maximum, which amounts to nearly † 2661″, it afterwards diminished, and towards 1775 was at its minimum, about ‡ 759″. The inclination of the orbit of this satellite to that of Jupiter, and the position of its nodes, are variable. These variations may be nearly represented by

* 27′ 58″ 9. † 14′ 22″ 1. ‡ 4′ 5″ 9.

supposing the orbit inclined about *2244″ to the equator of Jupiter, and assigning to its nodes a retrograde motion on the plane of this equator, in a period of 137 years.

The orbit of the fourth satellite has a very sensible ellipticity, its perigee has a direct motion of about †7852″. This ‡272″ to the orbit of Jupiter. It is in consequence of this inclination that the fourth satellite frequently passes behind the planet relatively to the Sun without being eclipsed. Since the discovery of the satellites, till 1760, its inclination has appeared constant, but it has augmented by a sensible quantity in these later years. We intend to return to these irregularities, when we shall explain their cause.

Independent of these variations the motions of the satellites of Jupiter are subject to inequalities which disturb their elliptic motions, and which render their

---

* 12′ 7″. † 42′ 24″. ‡ 1′ 28″ 1.

theory extremely complicated. These are principally sensible in the three first satellites, the motions of which present very remarkable relations.

Their mean motions are such that the motion of the first satellite plus twice that of the third, is nearly equal to three times the mean motion of the second satellite. The same proportion exists between their mean synodical motions; for the synodical motion being nothing more than the excess of the sidereal motion of a satellite above that of Jupiter, if we substitute the synodical instead of the mean motion, in the preceding equality, the mean motion of Jupiter disappears, and the equality remains the same.

The mean longitudes, whether synodical or sidereal, of the three first satellites seen from the centre of Jupiter, are such that the motion of the first satellite *minus* three times that of the second, *plus* twice that of the third, is nearly equal to the semicircumference. This equality is so near, that we are tempted to consider it as ri-

gorous, and to reject as errors of observation, those very small quantities by which they differ from it. We may at least be assured that it will continue through a long succession of ages, from which it results that from this time to an immense number of years, the three first satellites of Jupiter cannot be eclipsed at the same time.

The periods and the laws of the principal inequalities of these satellites are the same. The inequality of the first advances or retards its eclipses *233″ in time, at its maximum. Comparing it motion with the respective positions of the two first satellites, it has been found that it disappears when these satellites, seen from the centre of the planet, are in opposition to the Sun at the same time; that it afterwards increases, and is the greatest possible when the first satellite, at the moment of its opposition, is †50° more advanced than the second:

---

* 1′ 15″. †45°.

that it is again annihilated when it is more advanced by *100°; beyond this, it takes a contrary sign, and retards the eclipses, it augments to †150° distance between the satellites, where it is at its negative maximum; it then diminishes and disappears at ‡200 distance: and that finally in the second half of its circumference it follows the same laws as in the first. It has been concluded from this that there exists in the motion of the first satellite of Jupiter an inequality of § 5258″ at its maximum, and proportional to the sine of double the excess of the mean longitude of the first satellite above that of the second; an excess equal to the difference of the mean synodical longitudes of the two satellites. The period of this inequality is not four days. But how, in the eclipses of the first satellite, does it transform itself into a period of ‖ 437ʲ 75? This is what we are going to explain.

---

* 90°. † 135°. ‡ 180°. § 28′ 2″. ‖ 437ᵈ 18ʰ.

Let us suppose that the two first satellites set out together, from their mean oppositions to the Sun. At every circumference which the first satellite describes in consequence of its mean synodical motion, it will be in the mean opposition. If we imagine a fictitious star, whose angular motion may be equal to the excess of the mean synodical motion of the first satellite, above twice that of the second. Then twice the difference of the mean synodical motions of the two satellites will be, in the eclipses of the first, equal to the multiple of its circumference *plus* the motion of the fictitious star. The sine of this last motion will be then proportional to the inequality of the first satellite in its eclipses and may represent it. Its period is equal to the duration of the motion of the fictitious star, a duration which from the mean synodical motions of the two satellites is 437.^{j}75. It is thus determined with greater precision than by direct observation.

The inequality of the second satellite

follows a law similar to that of the first, with this difference, that it has always a contrary sign. It accelerates or retards the eclipses * 1059″ in time at its maximum. Comparing it with the respective positions of the two first satellites, it is observed that it disappears when they are at the same time in opposition to the Sun. That it then retards more and more the eclipses of the second satellite, till the two satellites are separated 100° from each other at the instant of these phenomena; that this retardation diminishes and becomes nothing when the mutual distance of the two satellites is 200°: finally, that beyond this term, the eclipses advance in the same manner as they before retarded. It has been concluded from these observations that an inequality of * 11923″ at its maximum exists in the motion of the second satellite, proportional (but affected by a contrary sign), to the sine of the excess of the mean longitude

---

* 5′ 43″. † 1° 4′ 23″.

of the first satellite above that of the second, an excess equal to the difference of the mean synodical motions of the two satellites.

If the two satellites set out together, from their mean opposition to the Sun, the second satellite will be in its mean opposition at each circumference which it will describe in consequence of its mean synodical motion. If we conceive, as before, a star whose angular motion may be equal to the excess of the mean synodical motion of the first satellite above twice that of the second, then the difference of the synodical motion of the two satellites will be, in the eclipses of the second, equal to a multiple of its circumference plus the motion of the fictitious star. The inequality of the second satellite, in its eclipses, will be then proportional to the sine of the motion of this imaginary star. We thus see the reason why the period and the law of this irregularity are the same as those of the inequality of the first satellite.

The influence of the first satellite upon

the inequality of the second, is very probable: but if the third satellite produced in the motion of the second, an inequality similar to that which the second seems to produce in the motion of the first, that is to say, proportional to the sine of double the difference of the mean longitudes of the second and third satellites; this new inequality would confound itself with that which is due to the first satellite; for in consequence of the relation which the longitudes of the three first satellites have to each other, and which we have described before, the difference of the mean longitudes of the two first satellites is equal to the semi-circumference plus double the difference of the mean longitudes of the second and third satellites; so that the sine of the first difference is the same with the sine of double the second difference with a contrary sign. The inequality produced by the third satellite in the motion of the second would therefore have the same sign, and follow the same law, as the inequality observed in this motion; it is therefore very

probable that this inequality is the result of two inequalities dependent on the first and third satellites. If by the succession of ages the relation between the mean longitudes of these three satellites should cease to exist, these two inequalities, at present confounded, would separate, and their respective value might be known. But according to observation this relation should subsist for a very long period, and we shall see in the fourth Book that it is rigorously exact.

Finally the inequality relating to the third satellite in its eclipses, compared with the respective positions of the second and third satellites, offers the same proportion as the inequality of the second compared with the respective positions of the two first satellites. There exists then in the motion of the third satellite, an inequality proportional to the sine of the excess of the mean longitude of the second satellite above that of the third, which inequality at its maximum is *827″.

* 4′ 27″ 9.

If we suppose a star whose angular motion may be equal to the excess of the mean synodical motion of the second satellite above twice the mean synodical motion of the third, the inequality of the third satellite will be, in its eclipses, proportional to the sine of the motion of this fictitious star. Now in consequence of the proportion which exists between the mean longitudes, the sine of this motion is, exclusive of the sign, the same with the motion of the first fictitious star which we considered. Thus the inequality of the third satellite, in its eclipses, has the same period, and follows the same laws, as the inequalities of the two first satellites.

Such is the motion of the principal inequalities of the three first satellites of Jupiter, which Bradley seems to have suspected, and which Wargentin has since investigated with the greatest accuracy.

Their correspondence and that of their mean motions and longitudes, seem to form a separate system of these three bo-

dies, animated by common forces, and united by common proportions.

Let us now consider the satellites of Saturn. If the semi-diameter of this planet seen at its mean distance from the Sun be taken as unity, the distances of the satellites from its centre will be as follows:

I.	-	3.080
II.	-	3.952
III.	-	4.893
IV.	-	6.268
V.	-	8.754
VI.	-	20.295
VII.	-	59.154

The durations of their sidereal revolutions are:

		J.	Days.	hrs.	min.	sec.
I.	-	0.94271	0	22	37	30
II.	-	1.37024	1	8	53	5.8
III.	-	1.88780	1	21	18	25.9
IV.	-	2.73948	2	17	44	36.6
V.	-	4.51749	4	12	25	11
VI.	-	15.9453	15	22	41	13.9
VII.	-	79.3296	19	7	54	27

Comparing the durations of the revolutions of these satellites, at their mean

distances from the centre of Saturn, we again find the beautiful proportion discovered by Kepler, relative to the planets, and which we have seen exist in the satellites of Jupiter, that is to say, *that the squares of the times of the revolutions of Saturn's satellites are to each other, as the cubes of their mean distances from the centre of this planet.*

The great distance of the satellites of Saturn and the difficulty of observing their positions, has not permitted us to discover the ellipticity of their orbits, and still less the inequalities to which their motions are subject; nevertheless the ellipticity of the orbit of the sixth satellite is perceptible.

If we take for unity the semi-diameter of Uranus supposed * 6″ seen at the mean distance of the planet from the Sun, the distances of its satellites will be :

I.	-	13.120
II.	-	17.022

---

*1″9.

III.	-	19.845
IV.	-	22.752
V.	-	45.507
VI.	-	91.008

The durations of their sidereal revolutions are:

		J.	Days.	hrs.	min.	sec.
I.	-	5.8926	5	21	25	20
II.	-	8.7068	8	16	57	47
III.	-	10.9611	10	23	2	47
IV.	-	13.4559	13	10	56	29
V.	-	38.0750	38	1	48	0
VI.	-	107.6944	107	16	39	56

These durations, with the exception of the second and fourth, have been concluded from the greatest observed elongations, and from the hypothesis that the squares of the sidereal revolutions of the satellites are as the cubes of their mean distances from the centre of the planet: an hypothesis which observation has confirmed regarding the second and fourth satellites of Uranus, so that it should be considered as a general law of the motion of a system of bodies round a common focus.

In the mean time what are the principal forces which retain the planets, satellites, and comets in their respective orbits? What particular forces disturb their elliptic motions? What cause produces the retrograde motion of the equinoxes? and moves the axes of rotation of the Earth and Moon? By what forces finally, are the waters of the ocean raised twice a day? The supposition of one sole principle on which all these effects depend, is worthy of the simplicity of nature.

The generality of the laws which the celestial motions present, seem to indicate its existence. Already even we may suspect this principle in the relation of these phenomena with the respective position of the bodies in the solar system. But to bring it forward with evidence the laws of the motion of matter must be known.

THE

SYSTEM OF THE WORLD.

BOOK III.

OF THE LAWS OF MOTION.

Surrounded by an infinite number of phenomena which continually succeed each other on the Earth, philosophers have succeeded in discovering the small number of general laws to which the motions of matter are subject. To them all nature is obedient; and every thing is derived from them as necessarily as the returns of the seasons; the curve described by the lightest atom that seems carried about by the winds as chance directs, is regulated by laws as certain as the planetary orbs.

The importance of these laws, on which we never cease to depend, ought to have excited the curiosity of mankind in all ages, but by the effect of indifference too common to the human mind, they were utterly unknown till the commencement of the last century, when Galileo first laid the foundation of the science of mechanics by his beautiful discoveries relative to the descent of falling bodies. Geometricians in following the steps of this great man, have finally reduced the whole science of mechanics to general formulæ, which leave nothing to be desired but to bring the art of analysis to perfection.

CHAP. I.

Of Forces and their Composition.

A BODY appears to us to be in motion when it changes its situation relatively to a system of bodies which we suppose to be in a state of repose. Thus in a vessel moving in a uniform manner bodies seem to us to move when they correspond successively to different parts of the vessel. This motion is only relative, for the vessel itself moves on the surface of the sea, which revolves round the axis of the Earth, the centre of which moves round the Sun, which is itself carried along the regions of space with the Earth and all the planets. To conceive a term to these motions, and to arrive at last at some fixed points from which we may reckon the absolute motion of bodies, we imagine a space without bounds, immovable and

penetrable to matter. It is to parts of this space, real or imaginary, that we refer in imagination the position of bodies, and we conceive them in motion when they correspond successively to different places in this space.

The nature of this singular modification by virtue of which a body is transported from one place to another is, and always will be, to us unknown. It has been designated by the name of FORCE; its effects and the law of its action is all that we can possibly determine.

The effect of a force acting on a material point is to put it in motion if nothing opposes it. The direction of the force is the straight line which it tends to make it describe. It is evident that if two forces act in the same direction they will increase the effect of each other; but if they act in opposite directions, the point will only move in consequence of their difference, and it would remain at rest if the forces were equal. If the directions of the two forces make an angle with

each other, the resulting force will take a mean direction, and it can be demonstrated by geometry alone, that if reckoning from the point of concourse of the two forces, we take on their directions, straight lines to represent them, and then form a parallelogram with these straight lines, its diagonal will represent both as to its direction and magnitude, their resulting force.

Thus the resulting force may be substituted for two composing forces, and reciprocally may be substituted for any force whatever, two others of which it is the resulting force. Any force, therefore, may be decomposed into two others parallel to two axes situated in the same plane, and perpendicular to each other. To do this it is sufficient to draw from the first extremity of the line representing the force, two lines parallel to these axis and to form with these lines a rectangle, the diagonal of which should be the force required to be decomposed. The two sides of this rectangle will represent the forces

into which the given force may be decomposed parallel to these axes.

If the force is inclined to a plane given in position, then, taking to represent it, a line in its direction, the extremity of which is in the surface of the plane, the perpendicular which falls from the other extremity on the plane, will be the primitive force decomposed in a direction perpendicular to the plane. The straight line which in the plane joins the force (or the other extremity of the line) with the perpendicular, will represent the primitive force decomposed parallel to the plane. This second partial force may be itself decomposed again into two others parallel to two axes in the same plane perpendicular to each other. Thus every force may be decomposed into three others parallel to three axes perpendicular to one another.

Hence arises a very simple method of having the resulting force, of any number of forces which act upon a material point, for by decomposing every one of them into three others parallel to three axes given in

position and perpendicular to each other, it is clear that all the forces parallel to the same axis will be reduced to one single force equal to the sum of the forces which act in one direction *minus* the sum of those which act in a contrary direction. Thus the point will be sollicited by three forces perpendicular one to the other; if then, three straight lines in each of their directions be taken to represent them, reckoning from the point of concourse, and on these three lines we form a rectangular parallelopipedon, the diagonal of this solid, will represent both the quantity and direction of the resulting force of all those which act upon the point.

CHAP. II.

Of the Motion of a Material Point.

A POINT in repose cannot give to itself any motion, since it contains not within itself any cause why it should move in one direction in preference to another. When sollicited by any force and then abandoned to itself, it will move constantly and uniformly in the direction of that force, if it meets with no resistance; that is at every instant the force and direction of its motion will be the same. This tendency of matter to persevere in a state of either motion or rest, is what is termed *inertia*, it is the first law of the motion of bodies.

The direction of motion in a straight line follows necessarily from this, that there is no reason why the point should deviate to the right, rather than to the left of its

primitive direction, but the uniformity of its motion is not equally evident. The nature of the moving force being unknown, it is impossible to know *à priori* if this force should preserve itself or not. It is true that since a body is incapable of giving itself any motion, it seems equally impossible that it should be able to effect any change in that which it has received, so that the law of inertia is at least the most natural and most simple that can be imagined. It is likewise confirmed by experience, for in fact we observe on the Earth that motions are perpetuated for a longer time in proportion as the obstacles which oppose them are diminished, and this would lead us to suppose that without these obstacles they would continue for ever. But the inertia of matter is principally remarkable in the motions of the heavenly bodies, which for a great number of ages have not experienced any sensible alteration. For these reasons we shall consider inertia as a law of nature, and when we observe any alteration in the

motion of a body, we shall conclude that it arises from the action of some foreign cause.

In uniform motion the spaces described are proportional to the times. But the times employed to describe a given space, are greater or less according to the magnitude of the moving force. From these differences has arisen the idea of velocity, which in uniform motion is the relation between the space and the time employed to describe it. To avoid comparing together, quantities so heterogeneous as space and time, we take an interval of time, a second for example, as an unity of time; and in like manner a portion of space, as a foot, for a unity of space. Time and space thus become abstract numbers, which express how many unities each contain of their own species, and thus they may be compared one with another. Velocity becomes the ratio of two abstract numbers, and its unity is the velocity of a body which describes one foot in one second. In reducing in this manner space,

time, and velocity to abstract numbers, it appears that space is equal to the product of the velocity by the time, which latter is consequently equal to the space divided by the velocity.

Force being only known to us by the space which it causes to be described in a given time, it is natural to take this space for its measure, but this supposes, that several forces acting in the same direction, will cause to be described in a unity of time, a space equal to the sum of the spaces which each would have caused to be described separately, or in other words that the space is proportional to the velocity. This is what we cannot be assured of, *à priori*, considering our ignorance of the nature of the moving force : upon this subject we must have recourse to experience, for whatever is not a necessary consequence of the few data we have given on the nature of things, is only to us the result of observation.

Force may be expressed by an infinity of functions of the velocity, without im-

plying a contradiction. There is none in supposing it proportional to the square of the velocity. In this hypothesis it is easy to determine the motion of a point sollicited by any number of forces, whose velocities are known; for if we take upon the directions of these forces, straight lines representing their velocities, beginning at their point of concourse, and if upon these directions, and from the same point other lines be taken, which are to each other as the squares of the first, these lines will represent the forces themselves. By combining them according to the rules already given, we shall obtain their resulting force, and also the straight line which represents it, and which will be to the square of the corresponding velocity, as the straight line representing one of the composing forces is to the square of its velocity. By this it appears how the motion of a point may be determined, whatever be the function of the velocity which expresses the force. Among all the func-

tions mathematically possible, let us examine which is that of nature.

It is observed upon the Earth that a body sollicited by any force, moves in the same manner, whatever be the angle which the direction of this force makes with the direction of the motion which is common to the body and to the part of the terrestrial surface to which it corresponds ; the same thing takes place in a vessel, whose motion is uniform ; a moveable body submitted to the action of a spring, or of gravity, or any other force, moves relatively to the parts of the ship in the same manner whatever be the velocity and direction of the vessel. It may then be established as a general law of terrestrial motions, that if in a system of bodies carried on by a common motion, any force be impressed on one of them, its apparent or relative motion will be the same, whatever be the general motion of the system, and the angle which its direction makes with the impelling force.

The proportionality of force to velocity, results from this law supposed rigorously exact; for if we suppose two bodies moving upon one straight line with equal velocities, and that by impressing on one of them a force, which encreases the primitive force, its relative velocity to the other body remains the same as if both of them had been primitively in a state of repose; it is evident that the space described by the body in consequence of its primitive force, and of that which is added to it, becomes equal to the sum of the spaces, which each of them would have caused it to describe in the same time; which supposes the force proportional to the velocity.

And reciprocally, if the force is proportional to the velocity, the relative motions of a system of bodies, animated by any forces whatever, are the same, whatever be their common motion, for this motion decomposed into three others, parallel to three fixed axes, only increases by the same quantity the partial velocities of each

body, parallel to these axes, and since the relative velocities only depends on the difference of these partial velocities, it will be the same, whatever be the motion common to all the bodies. It is therefore impossible to judge of the absolute motion of a system of which we make a part, by any appearances that can be observed. Which circumstance characterizes this law, the ignorance of which has so long retarded our knowledge of the system of the world, by the difficulty of conceiving the relative motions of projectiles about the surface of the Earth, carried along by a double motion round its own axis and round the Sun.

But considering the smallness of the most considerable motions which we can impress on bodies on the Earth, compared with that which they share in common with the Earth, it is sufficient for the appearances of a system of bodies to be independant of the direction of this motion that a small augmentation of the force by which the earth is animated, be to the

corresponding increase in its velocity, in the ratio of these quantities themselves. Thus our experiments only prove the reality of this proportion, which if it existed, whatever were the velocity of the Earth, would give the law of the velocity proportional to the force. It would give likewise this law, if the function of the velocity expressing the force, was only composed of a single term. If then the velocity be not proportional to the force we must suppose in nature the function of the velocity expressing the force to consist of several terms, which is not at all probable. We must moreover suppose the velocity of the Earth to be exactly such as suits the above proportion, which is likewise extremely improbable. Besides the velocity of the Earth varies in different seasons of the year, it is about one-thirtieth greater in winter than in summer: this variation is still more considerable, if, as every thing seems to indicate, the solar system itself is in motion in space, for according as this progressive motion con-

spires with that of the Earth, or is contrary to it, there should result great variations in the course of the year in the absolute motion of the Earth, and this should alter the proportion of which we are speaking, and the relation of the force impressed to the relative velocity resulting from it, if this proportion and this relation were not independant of the motion of the Earth. Nevertheless the most precise experiments have never shewn the slightest sensible variation.

All the celestial phenomena serve to corroborate these proofs. The velocity of light, determined by the eclipses of Jupiter's satellites, combines itself with that of the Earth, exactly according to the law of the proportionality of the force to the velocity, and all the motions of the solar system, calculated after this law, are entirely conformable to observation.

Hence we have two laws of motion, that is to say, inertia, and that of the force proportional to the velocity, given by observation; they are the most simple and

most natural that [illegible] without doubt [illegible] very nature of matter; but [illegible] being to us unknown, [illegible] only observed facts. They are the only ones that the science of mechanics borrows from experience.

The velocity being proportional to the force, these two quantities may be represented one by the other; and in the following rules we can always determine the velocity of a point sollicited by any number of forces, whose directions and intensities are known.

If the point is sollicited by a number of forces which act in a continued manner it will describe with a motion uniformly variable, a curve, the nature of which will depend on the laws by which the point is sollicited. To determine it we must consider the curve in its elements, examine how they arise one from the other, and ascend from the law of the augmentation of their ordinates to their finite expression. It is here that the infinitesimal calculus

spires with that of the Earth, or is contrary to it, there should result great variations in the course of the year in the absolute motion of the Earth, and this should alter the proportion of which we are speaking, and the relation of the force impressed to the relative velocity resulting from it, if this proportion and this relation were not independant of the motion of the Earth. Nevertheless the most precise experiments have never shewn the slightest sensible variation.

All the celestial phenomena serve to corroborate these proofs. The velocity of light, determined by the eclipses of Jupiter's satellites, combines itself with that of the Earth, exactly according to the law of the proportionality of the force to the velocity, and all the motions of the solar system, calculated after this law, are entirely conformable to observation.

Hence we have two laws of motion, that is to say, inertia, and that of the force proportional to the velocity, given by observation; they are the most simple and

most natural that can be imagined, and without doubt they are derived from the very nature of matter; but this nature being to us unknown, these laws are to us only observed facts. They are the only ones that the science of mechanics borrows from experience.

The velocity being proportional to the force, these two quantities may be represented one by the other, and by the preceding rules we can always determine the velocity of a point sollicited by any number of forces, whose velocities and directions are known.

If the point is sollicited by a number of forces which act in a continued manner, it will describe with a motion incessantly variable, a curve, the nature of which will depend on the forces by which the point is sollicited. To determine it we must consider the curve in its elements, examine how they arise one from the other, and ascend from the law of the augmentation of their ordinates to their finite expression. It is here that the infinitesmal calculus

becomes indispensible, and that we become sensible of the utility of bringing to perfection this powerful engine of the human mind.

We have in the instance of gravity, a daily example of a force, which seems to act without interruption. It is true we are ignorant whether its successive actions are separated by intervals of time, whose duration is insensible, but the phenomena being nearly the same either in this hypothesis or that of a continued action, geometricians have adopted the former as the most simple and commodious. Let us investigate the laws of these phenomena. Gravity seems to act upon bodies in the same manner whether they are in a state of repose or motion. In the first instant a body abandoned to its action acquires a degree of velocity infinitely small, in the second instant a new degree of velocity is added to the first, and so continues, the velocity increasing in proportion to the time.

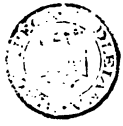
If we imagine a right-angled triangle,

and one side to represent the time and increase with it, the other side may represent the velocity. The element of the surface of the triangle, being equal to the product of the element of the time by the velocity, it will represent the element of the space which gravity causes to be described; and thus this space will be represented by the intire surface of the triangle, which increasing as the squares of one of its sides, shews us, that in motion accelerated by the action of gravity, the velocities increase as the times and the heights from which bodies fall fall from a state of rest, vary as the squares of the times, or as the squares of the velocities. In expressing thus by unity the space a body falls in one second, it will descend four unities in two seconds, nine in three seconds and so on; in such a manner that in each second it will describe spaces increasing as the odd numbers 1, 3, 5, 7, &c.

The space which a body will describe in consequence of the velocity acquired by its fall in a time equal to that of its de-

scent, will be the product of this time by its velocity, but this product is the double of the surface of the triangle, therefore a body moving uniformly with its acquired velocity will describe in the time equal to that of its descent a space double to that through which it has fallen.

The ratio of the required velocity to the times is constant for the same accelerating force, it augments and diminishes as they are greater or less, it may therefore serve to express them. The double of the space described being the product of the time by the velocity, the accelerating force is equal to this double space divided by the square of the times. It is likewise equal to the square of the velocity divided by this double space. These three methods of expressing the accelerating force are useful on many occasions, they do not give the absolute values of these forces, but their ratios to each other or to one of them, taken as unity, which is all that is required in mechanics.



On an inclined plane, the action of gravity is decomposed into two, one perpendicular to the plane which is destroyed by its resistance, the other parallel to the plane which is to the primitive force of gravity as the height of the plane is to its length ; motion is therefore uniformly accelerated on inclined planes ; but the velocities, and the spaces described, are to the velocities and spaces described in the same time in the direction of the vertical, as the height of the plane to its length. From hence it follows that all the chords of a circle which terminate in one of the extremities of a vertical diameter, are described in the same time by the action of gravity as the diameter itself.

A body projected in the direction of any straight line whatever, continually deviates from this direction, describing a curve concave to the horizon, and of which this line is the first tangent. Its motion when referred to this line by vertical lines, is uniform, but it accelerates in the direction of these verticals according to the

laws already explained. If, therefore, vertical lines are drawn from every part of the curve to this first tangent, they will be proportional to the squares of the coresponding parts of this tangent, a property which characterizes the parabola. If the force of projection is in the direction of the vertical itself, the parabola is confounded with this vertical line, and thus the formulæ for parabolic motion, give those for accelerated and retarded motion in the direction of the vertical.

Such are the laws of the descent of the heavy bodies, as discovered by Galileo. It appears now to us not difficult to have discovered them, but as they escaped the investigation of philosophers notwithstanding the phenomena which incessantly occurred it must have required an extraor- genius to have developed them.

We have seen in the first Book that a material point suspended at the extremity of a straight line without mass, and fixed at its other extremity, forms the simple pendulum. This pendulum when removed

from its vertical position, tends by its gravity to return, and this tendency is very nearly proportional to its deviation, when that is not very considerable. Let us imagine two pendulums of the same length quitting the vertical position at the same instant with very small velocities; they will describe, in the first instant, arcs proportional to the velocities; at the beginning of the second instant equal to the first, the velocities will be retarded in proportion to the arcs described, and to the primitive velocities. The arcs described, therefore, in this instant, will be proportional to these velocities; it will be the same with the arcs described in the third and fourth instants, &c. Thus at every instant the velocities and the arcs measured from the vertical, will be proportional to the primitive velocities, the pendulums will arrive, therefore, at the same moments at a state of repose; they will return again to the vertical by a motion accelerated according to the same laws by which their velocities had been retarded,

and will arrive at the same instant, and with their primitive velocity, they will oscillate in the same manner on the other side of the vertical, and would continue to oscillate for ever but for the resistance which they experience. It is evident that the extent of their oscillations is in proportion to their primitive velocities, but the length of their oscillations is the same and consequently independent of their extent. The accelerating force of the pendulum, not being exactly in proportion to the extent of the arc from the vertical, this isochronism is only approximative, relatively to a body performing very small oscillations in a circle. But it is rigorously exact in a curve, in which the force of gravity, decomposed parallel to the tangent, is proportional to the arc, reckoned from the lowest point of the curve, which immediately gives it differential equation. Huygens, to whom we are indebted for the application of pendulums to clocks, was interested in finding this curve, and in the manner of making pen-

dulums describe it. He found that it was a cycloid placed vertically, so that its summit might be the lowest point, and that to cause a body suspended at the extremity of an inextensible thread to describe this curve, he found that it would be necessary to fix the other extremity at the common origin of two cycloids, equal to that to be described by the pendulum, and so placed that the thread in its vibrations may envelope alternately a portion of these curves. But whatever ingenuity may have been manifested in these investigations, experience has preferred the circular pendulum, as the most simple and sufficiently exact in practice, but the theory of evolutes which has been derived from them, is become very important by its application to the system of the world.

The length of the very small oscillations of a circular pendulum, is to the time which a body will employ in falling from a height equal to double the length of the pendulum, as the semi-circumfe-

rence is to the diameter : and thus the time of the descent of a body through a small arc of a circle terminated in a vertical diameter, is to the time of the descent through the diameter, or what is the same thing, as the time required to describe the chord of this arc, as the quarter of the circumference is to the diameter, the straight line joining the two points is not therefore the line of swiftest descent from one point to another; the investigation of this line has excited the attention of mathematicians, and they have found that it is a cycloid whose origin is the most elevated point.

The length of a simple pendulum vibrating seconds, is to double the height which bodies describe by the force of gravity in one second, as the square of the diameter to the square of the circumference.

It has been observed in the First Book, that very exact experiments have given the length of the pendulum vibrating decimal seconds at Paris 2.28386, from which it results that bodies fall by the force of gravity 11^{f}.2704 in the first se-

cond of time. The connection between the lengths of pendulums, which may be observed with great precision, and the time of descent of falling bodies was an ingenious discovery for which we are indebted to Huygens.

The times of the very small oscillations of pendulums of different lengths and animated by the same force of gravity, are as the square roots of these lengths. If the pendulums are of the same length, but animated by different forces, the times of their oscillations will be reciprocally as the square roots of these forces.

It is by means of these theorems that variation of the force of gravity at the surface of the Earth, and on the summits of mountains has been determined. The observations made on pendulums have demonstrated that gravity neither depends on the figure nor on the surface of bodies, but that it penetrates into all their component particles, and tends to impress on them equal velocities in equal times. To be assured of this, Newton made several

bodies oscillate, of the same weight but different in their figure and matter, placing them in the interior of the same surface that the resistance in the air might be the same. Whatever precision he employed in the experiment, he was never able to perceive the smallest difference in the times of the vibrations of these bodies, from which it follows that were it not for the resistances which different bodies experience, their velocities acquired by the action of gravity, would always be the same in equal times.

We have likewise in circular motion an instance of force acting in a continued manner. The motion of matter abandoned to itself being uniform and rectilinear, it is evident that a body moving on the circumference of a circle tends at every instant to deviate from the centre in the direction of the tangent. The effort which it makes to this effect, is called the centrifugal force, and on the contrary every force directed to the centre is called a central or centripetal force; in a circular

motion the centripetal force is equal and directly contrary to the centrifugal force; it incessantly tends to draw the body towards the centre, and in an extremely small interval of time, its effect may be measured by the versed sine of the arc described.

We may by this result compare the force of gravity with the centrifugal force arising from the rotation of the Earth; at the equator bodies describe, in consequence of this rotation, an arc of 40.″1095 of the circumference in one second of time, the radius of this equator being 19634778 feet very nearly. The versed sine of this arc is 0.0389704. The force of gravity causes bodies to descend, at the equator, 11.23585 in one second; thus the central force necessary to retain these bodies at the surface of the Earth, and consequently the centrifugal force due to its motion, is to the force of gravity at the equator as 1. to 288.3. The centrifugal force diminishes the force of gravity, and bodies descend to the Earth at the equator

by the difference only between these two forces.

Calling then gravity the entire weight which would subsist without this diminution, the centrifugal force at the equator is $\frac{1}{289}$ of gravity. If the rotation of the Earth were seventeen times more rapid, the arc described at the equator in one second, would be seventeen times greater, and its versed sine would be 289 times more considerable, the centrifugal force would therefore be equal to that of gravity, and bodies at the equator would cease to gravitate towards the Earth.

In general the expression of a constant accelerating force acting always in the same direction, is equal to double the space it causes the body to describe, divided by the square of the time. Every accelerating force in an interval of time extremely short, may be considered as constant and acting in the same direction; moreover the space which is described by the action of the central force in circular motion, is the versed sine of the arc de-

scribed, which is nearly equal to the square of the arc divided by the diameter, the expression of this force is therefore the square of the arc described divided by the square of the time and by the radius of the circle. But the arc divided by the time is the velocity of the body, the central and likewise the centrifugal force are each therefore equal to the square of the velocity divided by the radius.

Let us compare this result with that found above, according to which gravity is equal to the square of the velocity acquired, divided by twice the space described; we shall see that the centrifugal force is equal to the force of gravity, even when the velocity of the revolving body is the same as that acquired by a heavy body, which should fall from a height equal to half the radius of the circumference described.

The velocities of several bodies moving in circles, are to each other as the circumferences which they describe, divided by the times of their revolutions, the circum-

ferences are as the radii, therefore the squares of the velocities are as the squares of the radii divided by the squares of the times. The centrifugal forces, therefore, are to each other as the radii of the circumferences divided by the squares of the times of revolution. Hence it follows that the centrifugal force, on the different terrestrial parallels, is proportional to the radii of these parallels. These beautiful theorems, discovered by Huygens, conducted Newton to the general theory of motion in curves, and to the law of universal gravitation.

A body describing a curve of any kind has a constant tendency to deviate in the direction of the tangent; now we may easily imagine a circle to pass through two adjacent elements of the curve, which is called the *circle of curvature* or the *osculatory circle* of that curve, in two consecutive instants the body may be conceived as moving in the circumference of this circle, its centrifugal force is therefore equal to the square of its velocity divided

by the radius of this circle of curvature; but the position and magnitude of this circle is constantly varying.

If the curve be described by virtue of a force directed to a fixed point, this force may be decomposed into two, one according to the radius of curvature, the other according to the element of the curve. The first is in equilibrio with the centrifugal force, the second augments or diminishes the velocity of the body, this velocity therefore continually varies. But it always is such that *the areas described by the radius vector about the origin of the force, are proportional to the times; and reciprocally, if the areas traced by the radius vector about a fixed point are proportional to the times, the force which sollicits the body will always be directed to this point.* These fundamental propositions in the theory of the system of the world, are easily demonstrated in the following manner:

The accelerating force may be supposed to act only at the commencement of each

instant during which the motion may be supposed to be uniform, the radius vector will thus describe a small triangle. If the force ceased to act in the following instant, the radius vector would trace in this new instant, a new triangle equal to the first, because the triangles having their vertex at the fixed point which is the origin of the force ; their bases situated on the same straight line will be equal, since they are described with the same velocity during two instants which are supposed equal, but at the commencement of this new instant, the accelerating force combining itself with the tangèntial force of the body, makes it describe the diagonal of a parallelogram whose sides represent these forces. The triangle which the radius vector describes in virtue of this combination of forces is equal to that which it would have described without the action of the accelerating force, for these triangles have for a common basis the radius vector at the end of the first instant, and the vertex of each is on a

straight line parallel to this basis. The areas, therefore, described by the radius vector, in the two consecutive instants, are equal to each other, and consequently the sector described by the radius vector increases as the number of these instants, or as the times. It is evident that this is only true inasmuch as the accelerating force is directed to one fixed point, otherwise the triangles which we have considered, will not have the same base and the same height: thus the proportionality of the areas to the times, demonstrates that the accelerating force is constantly directed to the origin of the radius vector.

In this case if a very small sector be imagined to be described during an extremely short instant of time, and from the first extremity of this sector, a tangent be drawn to the curve, and that the radius vector drawn from the origin of the force to the other extremity of the sector, be prolonged till it meet this tangent, then the part of this radius intercepted between this curve and the tangent, will evidently

be the space described by the central force.

In dividing the double of this space by the square of the time, we obtain an expression for this force; but the sector is proportional to the time, the central force, therefore, is, as the part of the radius vector intercepted by the curve and the tangent, divided by the square of the sector; strictly speaking, the central force in different points of the curve is not exactly equal to these quotients, but it approaches the nearer to them as we take the sectors very small, so that it is exactly equal to the limits of these quotients. The differential analysis gives this limit in a function of the radius vector, when the nature of the curve is known, and then that function of the distance is obtained to which the central force is proportional.

If the law of the force is given, the investigation of the curve described presents greater difficulties. But whatever be the nature of the forces by which a body is animated, the elementary variations of its

motion may easily be determined in the following manner :

Let us imagine three fixed axes, perpendicular among themselves, the position of a body may be determined at any instant, by three co-ordinates parallel to these axes, in decomposing every one of the forces which act upon the point, into three others parallel to these axes, the product of the resulting force of all these forces, parallel to one of these co-ordinates, by the element of the time during which it acts, will express the increase of the velocity of the body, in the direction parallel to that co-ordinate ; now this velocity, during this element, may be considered as uniform and equal to the element of the co-ordinate divided by the element of the time ; the elementary variation, therefore, of this quotient, is equal to the preceding product. The consideration of the other two co-ordinates affords two similar equations ; thus the determination of the motion of a body becomes an investigation of pure analysis, which is reduced to the in-

tegration of three differential equations. This integration is easy when the force is directed to a fixed point, but often the nature of forces renders it impossible; nevertheless the consideration of the differential equations leads to some interesting principles of mechanics, such as the following: The elementary variation of the square of the velocity of a body, submitted to the action of any accelerating forces whatever, is equal to double the sum of the products of each force, by the small space which the body advances in the direction of that force in a small instant of time: from which it may be concluded that the velocity acquired by a heavy body, descending along a line or curved surface, is the same as if it had fallen vertically from the same height.

Many philosophers, struck with the order which prevails in nature, and with the fecundity of the means by which phenomena are produced, have concluded that she always accomplishes her ends by ways the most simple.

In extending this idea to mechanics, they have sought what was the economy of nature in the employment of forces, and after many fruitless attempts, they at last discovered that of all the curves that a body might describe in going from one point to another, it always chose that in which the integral of the product of the mass of the body by its velocity, and by the element of the curve, is a *minimum*. Thus the velocity of a body moving in a curved surface and not solicited by any force being constant, it proceeds from one point to another by the shortest line. The preceding integral is called the *action of a body*, and the re-union of similar integrals, relative to the bodies of a system is called the *action of the system*. According to these philosophers the economy of nature consists in sparing this action, so that it may always be the least possible. It is this which constitutes *the principle of the least action*. But this principle is nothing but a curious result of the primordial laws of motion, laws, which as we

have seen, are the most simple and natural that can be imagined, and which seem to be derived from the essence of matter itself. All the laws, mathematically possible, offer analogous results, it ought not therefore to be elevated to the dignity of a final cause, for so far from having given birth to the laws of motion, it has not even contributed to their discovery, without which we should still dispute upon what was to be understood by the least action of nature.

CHAP. III.

Of the Equilibrium of a System of Bodies.

The most simple case of equilibrium between several bodies, is that of two material points meeting each other with equal and opposite velocities. Their velocities will be destroyed, and the points reduced to a state of rest, by their mutual impenetrability, a property of matter by which two bodies cannot at the same moment occupy the same point of space. But if two bodies, of different masses, meet with opposite velocities, what then is the relation of their velocities to their masses, in case of equilibrium? To solve this problem, let us suppose a system of material points, contiguous to each other, arranged upon the same straight line, and impelled by a common velocity in the direction of this

straight line; let us suppose also a second system of contiguous material points arranged upon the same straight line, having a common velocity, but in a direction contrary to the first system, so that the two systems, after mutually striking each other, may remain in equilibrio. It is clear that if the first system was only composed of a single material point, every point in the second system would destroy in the striking point a part of its velocity equal to the velocity of this system; in the case of equilibrium, therefore, the velocity of the striking point should be equal to the product of the velocity of the second system by the number of its point, and we may substitute for the first system a single point impelled by a velocity equal to this product. For the second system likewise may be substituted a material point impelled by a velocity equal to the product of the velocity of the first system by the number of its points. Therefore instead of the two systems we shall have two points that will sustain each other in equi-

librio with contrary velocities, of which one will be the product of the velocity of the first system, by the number of its points, and the other will be the product of the velocity of the points of the second system, by their number; these products, therefore, in the case of equilibrium, should be equal to each other.

The mass of a body is the sum of its material points. The product of the mass by the velocity is called the *quantity of motion*, the same thing is understood of the *force of a body*. In the case of equilibrium between two bodies, or of two systems of material points that strike each other in contrary directions, the quantities of motion, or the forces opposed to each other should be equal, and consequently these velocities should be proportional to the masses.

Two material points cannot act on each other but in the direction of the straight line that joins them, the action which the first exerts upon the second communicates to it a quantity of motion; now the second

may be conceived to be sollicited by this quantity, and by another equal and directly opposite; the action of the first body is therefore entirely employed in destroying this last quantity of motion; but to do this it must employ a quantity of motion equal and contrary to that which is to be destroyed. Hence it appears that in the mutual action of bodies, action and reaction are always equal and in contrary directions. We see likewise that this equality does not suppose any particular force to reside in matter, but results from the impossibility that a body should acquire motion from the action of another without depriving it of a portion, in the same manner as a vase can only be filled at the expence of another which communicates with it.

The equality of action and reaction manifests itself in all the operations of nature. Iron attracts the loadstone in the same manner as it is itself attracted, the same is observed in electric attractions and repulsions, in the developement of elastic forces,

and even in the forces of animals, for whatever may be the nature of the motive power of man, and of animals, it is clear they receive by the reaction of matter a force equal and contrary to that which they communicate; and thus in this respect they are subject to the same laws as inanimate beings.

The reciprocity of velocity and mass in the case of equilibrium, serves to determine the ratio of the masses in different bodies. Those of homogeneous bodies are proportional to their volumes, which geometry teaches us to measure; but all bodies are not homogeneous, and the differences which exist either in their integrant particles, or in the number and magnitude of the intervals or pores which separate these particles, produce very great ones in the masses which are contained under the same volume. Geometry then becomes insufficient to determine the ratio of these masses, and we are obliged to have recourse to the science of mechanics.

If we conceive two globes of different

matter. If two globes be taken of different materials, whose diameters be supposed to vary till they become in equilibrio with each other when impelled by equal and contrary velocities, we may be assured that then, they each contain an equal number of material points, and consequently that their masses are equal. The ratio which the volumes of these substances bears to their equality of mass will thus be obtained, and afterwards we may determine by geometry the ratio of the masses of any two volumes whatever of the same substance. But this method would be excessively laborious, for the numberless comparisons which the wants of society require for the purposes of commerce; fortunately nature offers us in the weight of bodies a very simple method of comparing their masses.

It has been shewn in the preceding chapter that every material point on the same part of the Earth's surface, tends to descend with the same velocity by the action of gravity. The sum of these tendencies is what constitutes the weight of

a body, and thus the weights are proportional to the masses. Hence it follows that if two bodies suspended by a thread passing over a pulley, are in equilibrio when an equal portion of the thread is on each side the pulley, the masses of these bodies are equal, since tending to move with the same velocity by the action of gravity, their action upon each other is the same as if they struck each other with equal and contrary velocities. Two bodies may also be placed in equilibrio by means of a balance, whose arms are perfectly equal, which will be a proof of the equality of their masses. The ratio between the masses of different bodies is likewise ascertained by means of an exact balance, a number of small equal weights, by observing how many of these weights are necessary to produce an equilibrium.

The density of a body depends on the number of its material points included in a given volume, it is therefore proportional to the ratios of the mass to the volume. If a body existed without pores its density

would be the greatest possible, and by comparing its density with that of other bodies we should be able to determine the quantity of matter they contained. But being ignorant of such a substance, we can only ascertain the relative density of bodies; these densities are in proportion to the weight under the same volume, since the weights are proportional to the masses; taking therefore as unity the density of any substance whatever, for instance distilled water at the temperature of melting ice, the density of another body, will be the ratio of its weight to that of an equal volume of water; and this is called its *specific gravity*.

What we have said, seems to suppose that matter is homogeneous, and that bodies only differ from each other in their figure, and by the magnitude of their pores and integrant particles, but it is possible that there may be essential differences in the very nature of these particles; but this is indifferent to the science of mechanics, which only considers bodies

with relation to their motions. We may therefore, without fear of error, admit the homogeneous nature of matter, provided that by equal masses, we understand masses, which being sollicited by equal and contrary velocities, produce an equilibrium.

In the theory of equilibrium and of the motion of bodies, we omit every consideration of the figure and of the number of the pores which they contain. But attention is paid to the difference of their respective densities, by supposing them formed of material points more or less dense, which in fluids are perfectly free, and which in hard bodies are united by inflexible straight lines without mass, and which in soft and elastic bodies are both flexible and extensive. It is evident that in these suppositions, bodies will offer the same appearances which they really present to us.

The conditions of equilibrium of a system of bodies may always be determined by the laws of the composition of forces,

explained in the First Chapter of this Book; for we may conceive the force by which every material point in the system is animated, to be applied to that point in its direction, in which are united the directions of those forces which destroy it, or which by combining with it, form a resulting force, which in the case of equilibrium is destroyed by the fixed points of the system. Let us consider, for example, two material points attached to the extremity of an inflexible lever, and suppose the forces which solicit them to be in the plane of the lever. In conceiving these forces united at the point of concourse of their directions, the resulting force arising from their combination, should, to produce an equilibrium, pass through the fulcrum, which can alone destroy it, and it appears that from the law of the composition of forces, that the producing forces are in this case reciprocally to each other as the perpendiculars drawn from the point of support or fulcrum to the direction of the forcer.

If we imagine two ponderous bodies attached to the extremities of a rectilinear and inflexible lever; whose mass is infinitely small, relatively to that of the bodies, the directions parallel to the force of gravity may be conceived to meet at an infinite distance, in this case the forces which solicit the bodies, or what is the same thing, their weights, to produce an equilibrium, must be reciprocally as the perpendiculars drawn from the point of support to the directions of the forces, but these perpendiculars are in proportion to the arms of the lever. Thus the weights of bodies in equilibrio are reciprocally as the arms of the lever to which they are attached.

A very small weight may therefore be made to sustain a very large one in equilibrio, by means of a lever and other machines which are referrable to the same principle; and by this method we can lift an enormous weight by a very slight effort, but for this purpose, the power

must describe a great space to elevate the weight a very small height, so that what is gained in force is lost in time, and this takes place in all machines. But time may often be disposed of at pleasure, when we can only employ a very limited force. In other cases where a great velocity is to be produced, it may be effected by means of a lever, by applying the force to the shortest arm; it is in this possibility of augmenting at pleasure either the mass or velocity of the moving power, that the principal advantage of machinery consists.

In examining attentively in a great number of cases the conditions of equilibrium of a system of bodies, and the relation of each force to the velocity of the body to which it is applied, when the equilibrium of the system begins to give way, the following principle has been recognized, which contains in the most general manner the conditions of equilibrium of a system of material points, solicited by any force whatever.

If the position of a system be changed

by an infinitely small quantity (in a manner consistent with the conditions of the connection of its different parts), every material point will advance in the direction of the force by which it is solicited, a quantity, equal to the part of this direction comprised between the first position of the point and the perpendicular drawn from the second position of the point upon this direction: that is, in the case of equilibrium, the sum of the products of every force, by the quantity which the point to which it is applied advances in its direction, is equal to nothing. In this consists the principle of virtual velocities, for which we are indebted to John Bernoilli; but in practice, it should be observed that the products must be taken negatively of those points which move in a direction contrary to their forces. It must be recollected also, that the force is the product of the mass of a material point by the velocity which it would receive if entirely free.

If we conceive the position of every

point of the system to be determined by three rectangular co-ordinates; the sum of the products of every force, by the quantity which the point which it solicits advances in its direction, when the position of the system is deranged by an infinitely small quantity, will be expressed by a linear function of the variation of all the co-ordinates of these points. These variations will have with each other certain relations, which will depend on the manner in which the different parts of the system are connected together. In reducing, therefore, by means of the conditions of this connection, the arbitrary variations to the least possible number, in the preceding sum, which should be zero in the case of equilibrium, we must, for the equilibrium to take place in every direction, make the coefficients of the remaining variations separately equal to zero, which will give as many variations as there are arbitrary variations. These equations combined with those which arise from the connection of the parts of the system will con-

tain all the conditions of its equilibrium. Let us consider, for example, a system of ponderable points connected together in an invariable manner, and suppose it attached to a fixed point, round which it can turn freely in all directions; let us imagine three axes passing through this fixed point, and perpendicular to each other, and the position of every point of the system determined by three coordinates parallel to these axes, and the direction of its gravity decomposed according to the direction of these co-ordinates.

If the system be now moved an infinitely small quantity round the first axis, it is evident that the quantity which every point advances in the direction of the force parallel to this axis is equal to nothing. The quantity that it advances in the direction of the force parallel to the second axis, is equal to the product of the angular motion of rotation of the system, by the co-ordinate parallel to the third axis; and the quantity which it advances in the direction of the force parallel to the

third axis, is equal (but with a contrary sign) to the angular motion of rotation of the system by the co-ordinate parallel to the second axis: thus, according to the preceding principle, the system should be in equilibrio about its first axis, when the sum of the products of the mass of every point by its force parallel to the second axis, and by the co-ordinate parallel to the third axis, is equal to the sum of the products of the mass of every point, by the force parallel to the third axis, and by the co-ordinate parallel to the second.

That the equilibrium may subsist [in every position of the system round the principal axis, it is necessary that the preceding equality should subsist, whatever may be the forces parallel to the second and third axes, since it must be independant of the direction of gravity relatively to these axes; each of these preceding sums should therefore be made equal to zero, and as the partial actions of gravity decomposed parallel to any axis whatever, are the same for every point of the system,

the sum of the products of each of these points, by its co-ordinate parallel to the second or third axis, is zero. If the same equality subsists relatively to the co-ordinates parallel to the first axis, it is easy to see that it will subsist equally, with respect to any other axis passing through the fixed point; hence it follows that whatever be the situation of the system round this point, it cannot turn round any axis, but will remain in equilibrio, in consequence of the action of gravity. The point which possesses this remarkable property is called the *centre of gravity* of the system, it is such that whatever be the position of the system, the resulting force of all the efforts of gravity pass constantly through it.

To determine it, we refer its position and that of the points of the system to three fixed axes perpendicular to each other, and one of the co-ordinates of this centre, multiplied by the entire mass of the system, is equal to the sum of the products of the mass of each point by its corresponding co-ordinate. Thus the determination

of this centre, of which gravity first suggested the idea, is independant of it. The consideration of this centre, extended to a system of bodies, whether ponderable or not, is of great use in the science of mechanics.

CHAP. IV.

Of the Equilibrium of Fluids.

It has been observed in the First Book, that elastic fluids, such as air, owe their origin to heat, and incompressible fluids, such as water, to the pressure of the atmosphere. But to determine the laws of their equilibrium, we have only to consider them as formed of an infinite number of particles perfectly movable among each other, and yielding to the smallest pression which is exerted on one side more than on the other.

It follows from this characteristic property of fluids, that the force that solicits every particle of the free surface of a fluid in equilibrio is perpendicular to this surface; gravity is consequently perpendi-

cular to the surface of stagnant waters, which is for this reason horizontal.

In virtue of the mobility of its parts a ponderable fluid may exercise a pressure much greater than its weight. A small column of water, for example, terminated by a large horizontal surface presses as much the base on which it reposes as a cylinder of water of the same base and height. To understand the truth of this paradox we may imagine a fixed cylindric vase, whose bottom or base is moveable, suppose this vase filled with water, and its base kept in equilibrio by a force equal and contrary to the pression it experiences, it is clear that the equilibrium would still subsist, even if a part of the fluid was to consolidate itself and to unite with the sides of the vessel; for, in general, the equilibrium of a system of bodies, is not deranged, by the supposition of several of them uniting, or becoming attached to fixed points; we may thus form an infinity of vases of different figures, having all the same base and the same height as the cy-

lindric vase, and in which the water will exercise the same pressure on the movable base.

The pression which a fluid exerts against any surface is perpendicular to each of its elements, otherwise the fluid particle contiguous to it would slide, by the decomposition of the pression which it experienced. If a fluid acts only by its weight, its entire pression is equal to the weight of a prism of the fluid, whose base is equal to the surface pressed, and whose height is the distance of the centre of gravity of this surface from the level surface of the fluid.

A body plunged in a fluid, loses a part of its weight, equal in weight to a volume of the displaced fluid ;—for before the immersion, the surrounding fluid formed an equilibrium to the weight of this volume of fluid, which without disturbing the equilibrium may be supposed to have formed itself into a solid mass, the resulting force of all the actions of the fluid upon this mass, should be in equilibrio

with its weight, and pass through its centre of gravity; now it is clear that these actions are the same upon a body which occupies its place, the action of the fluid destroys therefore a part of the weight of the body equal to the weight of the volume of the displaced fluid.

Thus bodies weigh less in air than in vacuo; this difference, though but small, is not to be neglected in delicate experiments.

By means of a balance which carries at the extremity of one of its arms a body which can be plunged in a fluid, we can measure very exactly the diminution of weight which the body experiences in this immersion, and determine its specific gravity, or its relative density compared with that of the fluid. This is the ratio of the weight of a body in vacuo to the diminution of its weight when it is entirely immersed in the fluid. It is thus that the specific gravities of bodies are determined, compared with distilled water.

For a body lighter than a fluid to be in

equilibrio at its surface, its weight must be equal to the volume of the displaced fluid; it is moreover indispensable that the centres of gravity of this portion of the fluid and of the body, be in the same vertical, for the resulting force of the actions of gravity upon all the particles of the body pass through its centre of gravity, and the resulting force of all the actions of the fluid upon the body, pass through the centre of gravity of the displaced fluid; these resulting forces, therefore, must be on the same line, since they destroy each other, the centres of gravity are therefore in the same vertical.

There exist two states of equilibrium entirely distinct. In one, if the equilibrium is ever so little deranged, the bodies which compose the system only oscillate about their primitive position, and the equilibrium is said to be firm or stable.

This stability is absolute if it takes place, whatever be the nature of the oscillations; it is relative if it only takes place in oscillations of a certain kind. In the

other state of equilibrium, if the system be deranged, all the bodies deviate more and more from their primitive state.

We may form a just idea of these two states by considering an ellipse placed vertically on an horizontal plane. If the ellipse is in equilibrio on its smaller axis, it is clear that when deranged a little from this situation, it tends to return, making oscillations which the friction and resistance of the air soon destroy; but if the ellipse be in equilibrio upon its greatest axis, upon being once deranged it will deviate more and more, and finish by reversing itself upon its lesser axis. The stability of an equilibrium, therefore, depends on the nature of the small oscillations which the system, upon being deranged, makes about its primitive position. Very often this investigation is attended with great difficulty; but in many cases, and particularly in that of floating bodies, it is sufficient to judge of the stability of an equilibrium to determine if the force which solicits the system when deranged from

this state, tends to restore it to the same state again. In bodies floating on water, or any other fluid, it may be determined by the following rule :

If through the centre of gravity of the section of the surface of the water, of a floating body, we conceive a horizontal axis, such, that the sum of the products of every element of the section, multiplied by the square of its distance from this axis, be less than relatively to any other horizontal axis drawn through the same centre ; the equilibrium will be stable in every direction, when this sum surpasses the product of the volume of the displaced fluid by the height of the centre of gravity of the body, above the centre of gravity of this volume.

This rule is principally useful in the construction of vessels which require sufficient stability to enable them to resist the efforts of storms which tend to submerge them. In a ship the axis drawn from the stern to the prow is that relatively to which the sum abovementioned is a minimum ; it is easy, therefore, to

ascertain and measure its stability by the preceding rule.

Two fluids contained in a vase, dispose themselves in such a manner that the heaviest occupies the lowest part of the vessel, and the surface which separates them is horizontal.

If two fluids communicate with each other by means of a bent tube the surface which separates them in a state of equilibrium is horizontal, and their heights above this surface are reciprocally as their specific densities. In supposing to the whole atmosphere the density of the air at the temperature of melting ice, and compressed by a column of mercury of two feet and a half, its height will be equal to 23690 feet; but because the density of the atmospheric strata diminish as they are elevated above the surface of the earth, the height of the atmosphere is much greater.

To trace the general laws of the equilibrium of a fluid mass solicited by any forces whatever, we shall observe that

every point in the interior of this mass suffers a pression which in the atmosphere is measured by the height of the barometer, and which may be done in a similar manner by any other fluid. In considering every particle as an infinitely small parallelopiped the pression of the surrounding fluid will be perpendicular to that of the parallelopiped, which will tend to move itself perpendicularly to every one of its faces, by virtue of the difference of the pressions on two opposite faces. From these different pressions arise three forces perpendicular to each other, which must be combined with the other forces which solicit the fluid particle. And since this particle should be in equilibrio, in consequence of all these forces, the principle of virtual velocities will give the general equations of its equilibrium, whatever may be the position of the entire mass. The conditions of the integrability of these differential equations will determine the relations that should exist between the forces by which the fluid is solicited, to render the

equilibrium possible : their integration will give the pression which every fluid particle is subject to, and this pression will determine its elasticity and density, if the fluid is elastic and compressible.

CHAP. V.

Of the Motion of a System of Bodies.

Let us consider first the action of two material points of different masses, and which, moving on the same straight line, meet and strike each other. We may conceive immediately before the shock their velocities to be decomposed in such a manner that they may have a common velocity, and two contrary velocities, such as that if they have these alone they would have remained in equilibrio. The velocity common to each of the points, is not altered by their mutual action, and therefore will subsist alone after the shock. To determine it we shall observe, that the quantity of motion of the two points in virtue of this common velocity *plus* the quantity of motion due to the velocities destroyed, represents the sum of the quantities of

motion before the shock, provided we take the quantities of motion due to contrary velocities with contrary signs; but by the condition of equilibrium, the sum of the quantities of the motion due to the velocities destroyed equals nothing; therefore the quantity of motion due to the common velocity, is equal to that which primitively existed between the two points, and consequently this velocity is equal to the sum of the quantities of motion divided by the sum of the masses.

To find the velocity after the shock when the points are perfectly elastic, we must add or subtract from the common velocity which they would have if they were unelastic, the velocity which they will acquire or lose according to this hypothesis: perfect elasticity doubles these effects by completely restoring the springs which the shock had compressed, the velocity of each point after the shock may therefore be obtained by subtracting its velocity before the shock from double the common velocity; from which it is

easy to infer that the sum of the products of each mass, by the square of the velocity is the same before as after the shock of the two points; and this is true universally in the shock of any number of bodies which are perfectly elastic, in whatever manner they act upon each other.

The shock of two material points is merely ideal; but it is easy to reduce to the same case the shock of any two bodies whatever by observing that if these bodies strike each other according to the straight line connecting their centres of gravity, and perpendicular to their surface of contact, they will act one upon the other as if their masses were united in these centres; therefore motion is communicated to them in the same manner as between two material points whose masses are respectively these bodies.

Such are the laws of communication of motion, laws which experience confirms, and which are derived mathematically from the two fundamental laws of motion explained in Chap. II. of this Book.

Several philosophers have endeavoured to determine them from the consideration of final causes. Descartes, persuaded that the quantity of motion in the universe should always preserve itself the same, has deduced from this false hypothesis, false laws of the communication of motion, offering an example of the errors to which we are exposed, when we attempt to guess at the laws of nature, by attributing to her particular views.

When a body receives an impulsion in the direction passing through the centre of gravity, all its parts move with an equal velocity. If the direction passes on one side of this point, the different parts of the body acquire unequal velocities, and from this inequality of velocity, there results a motion of rotation of the body about its centre of gravity, at the same time that this centre is carried on with the velocity that it would have taken if the direction of impulsion had passed through this point. This case is that of the Earth and planets. To explain the double

motion of rotation and translation of the Earth, it is sufficient to suppose that it received a primitive impulsion in a direction passing at a small distance from its centre of gravity; and this distance in the hypothesis of the homogenity of this planet, is nearly the $\frac{1}{160}$th part of its radius. It is infinitely improbable that the primitive impulsion of the planets should have been in the directions of their centres of gravity, all the celestial bodies therefore should turn upon themselves. For the same reason the Sun, which turns on itself, must have received an impulsion which, not passing through its centre of gravity, carries it along through space with the planetary system, except an impulsion in a contrary direction should have totally destroyed this motion, which is not at all probable.

The impulsion given to a homogeneous sphere, which does not pass through its centre, causes it constantly to turn round a diameter which is perpendicular to a plane passing through its centre, and the

direction of the force impressed. New forces which solicit all its points, and whose resulting force passes through the centre, do not alter the parallelism of its axis of rotation.

It is thus that the axis of the Earth remains always very nearly parallel to itself in its revolution round the Sun, without any occasion to suppose with Copernicus an annual motion of the poles of the Earth, round those of the ecliptic. If the body be of any figure whatever, its axis of rotation may vary at every instant. The investigation of these variations is the most interesting problem relating to hard bodies, by its connection with the precession of the equinoxes, and the libration of the Moon. Its solution has led to this curious and useful result, namely, that in every body there exist three axes, perpendicular to each other, round which it may revolve uniformly when not solicited by external forces ; these axes have, for this reason, been called the principal axes of rotation.

A body, or a system of ponderable bodies, of any figure, oscillating about a fixed and horizontal axis, forms a compound pendulum. There exists no other in nature, and the simple pendulum of which we have spoken above, is an ideal geometrical conception for the purpose of simplifying the subject. To this it is easy to refer compound pendulums, in which all the points are firmly united together.

If we multiply the length of a simple pendulum, whose oscillations are of the same length as a compound pendulum, by the entire mass of this latter pendulum, and by the distance of its centre of gravity from its axis of oscillation, the product will be equal to the sum of the products of every particle of the compound pendulum by the square of its distance from the axis of oscillation. It is by this rule discovered by Huygens, that from experiments on compound pendulums the length of the simple pendulum has been determined which vibrates seconds.

Let us imagine a pendulum making very small oscillations, and at the moment when it deviates the most from the vertical, a small force to be impressed on it in a direction perpendicular to the plane of its motion; it will describe an ellipse round the vertical. To represent this motion we may suppose a fictitious pendulum which continues to vibrate as this would have done if it had not received this new impulsion, and at the same time that this latter pendulum vibrates on each side the imaginary one as if this pendulum was immovable and vertical, and thus the motion of the real pendulum is the result of two simple oscillations which exist together, and which it is easy to determine.

This method of considering the small oscillations of bodies, may be extended to any system whatever. If the system be supposed deranged from its state of equilibrium by very small impulsions, and afterwards to receive new impulsions, it will then oscillate, with respect to the successive situations which it would have taken by virtue of its first impulsions, in the

same manner as it would oscillate relatively to its state of equilibrium, if these new impulsions had been the only ones impressed on this state. Thus the very small oscillations of a system of bodies, however complicated, may be considered as performed by simple oscillations, perfectly similar to those of the pendulum. For if we conceive the system to be a little deranged from its state of equilibrium, in such a manner that the force soliciting each body of the system may be directed towards the point which it occupied in this state, and at the same time proportional to its distance from this point; it is evident that this will be the case during the oscillation of the system, and that the velocity of each body will be proportional to its distance from a state of equilibrium. They will arrive, therefore, at the same instant at this position, and will oscillate in the same manner as the simple pendulum. But the state of derangement which we have supposed to the system is not *unique*. If we cause one of

the bodies to deviate from its state of equilibrium, and then investigate the position of the remaining bodies, which may satisfy the preceding conditions, an equation is obtained of a degree equal to the number of bodies in the system, which will give as many simple oscillations as there are bodies. Let us conceive the first of these oscillations to exist in the system, and that at any instant, all the bodies of the system deviate from their position, proportionally to the relative quantities of the second simple oscillation. In consequence of the co-existence of these oscillations, the system will oscillate, relatively to the successive positions which it would have taken in consequence of the first simple oscillation, in the same manner as it would have oscillated about its state of equilibrium in consequence of the second oscillation. Its motion will therefore be composed of the two first simple oscillations. The third oscillation may be combined with these in the same manner, and by continuing thus to combine the rest,

we may represent all the motions of the system in the most general manner.

And hence results an easy method of determining the absolute stability of its equilibrium. If in all the positions relative to each simple oscillation, the forces which solicit the bodies tend to bring them back to their state of equilibrium, it is then stable, it will be otherwise or will only have a relative stability if in some one of its positions, the forces tend to make the bodies deviate from it.

In the waves we have a sensible example of the co-existence of small oscillations. When a point in the surface of stagnant water is slightly deranged, we see circular waves form and extend around it; by agitating the surface in another point, new waves are formed which mingle with the first and they dispose themselves over the surface, already agitated by the first waves, as they would have done if this surface had been entirely tranquil; so that each remains distinctly perceptible when thus united. That which the eye distinguishes

in the waves of water, the ear also perceives in sounds or undulations of the air, which are propagated simultaneously without alteration, producing very distinct impressions.

The principle of the co-existence of simple oscillations which we owe to Daniel Bernouilli, is one of those general results which is interesting from the facility which it gives to the imagination of representing phenomena and their successive changes. It may easily be deduced from the analytic theory of the small oscillations of a system. They depend on differential linear equations whose complete integrals are the sum of the particular integrals. Thus the simple oscillations dispose themselves one upon the other to form the motion of the system. It is interesting to follow in this manner the intellectual truths of analysis, in the phenomena of nature. The system of the world affords numberless instances of a similar correspondence, and it is this which constitutes the greatest charm of mathematical speculations.

It seems natural to reduce the laws of the motion of bodies to some general principle, in the same manner as the laws of their equilibrium have been reduced to the principle of virtual velocities. To effect this, let us consider the motion of a system of bodies acting one upon another without being solicited by accelerating forces. Their velocities change at every instant, but we may conceive each velocity at any instant as composed of that which takes place in the following instant, and of another velocity which should be destroyed at the beginning of this second instant. If the velocity destroyed be known, it will be easy to calculate the velocity in the second instant by the law of the decomposition of forces: now, it is clear that if these bodies had only been animated by the velocities destroyed, they would mutually have maintained themselves in a state of equilibrium. Thus the laws of equilibrium will give the ratios of the velocities destroyed, and it will be easy

to conclude the velocities which remain and their directions. The infinitesmal calculus will then give the successive variations in the motion of the system, and its position at every instant. It is evident that if the bodies are animated by accelerating forces, the same decomposition of velocities may be employed, but then the equilibrium must subsist between the destroyed velocities and these forces.

This method of reducing the laws of motion to those of equilibrium, and for which we are principally indebted to D'Alembert, is very general and luminous. And we should be surprised that it had escaped the notice of former geometricians, who had occupied themselves on the principles of dynamics, if we did not know how often it happens that the most simple ideas offer themselves the last to the human mind.

It remained only to combine the principle just explained with that of virtual

velocities, to give to the science of mechanics all the perfection of which it is susceptible.

This is what Lagrange has accomplished, and by this means has reduced the investigation of the motion of any system of bodies to the integration of differential equations; the object of mechanics is thus accomplished, and it belongs then to pure analysis to finish the solution of the problem. The most simple manner of forming these equations is this:

If we imagine three fixed axes perpendicular to one another, and if at any instant the velocity of every material point of a system of bodies be decomposed into three others parallel to these three, every partial velocity may be considered as uniform during this instant, and *we may conceive the point to be solicited at the end of this instant,* in a direction parallel to one of these axes, by three velocities, namely, by the velocity in this instant, by the little variation it receives in the following in-

stant, and by this same variation applied in a contrary direction. The two first of these velocities subsist in the following instant, the third should be destroyed by the forces which sollicit the point, and by the action of the other points of the system: thus if we suppose the instantaneous variations of the partial velocities of every point of the system to be applied to this point in a contrary direction, the system should remain in equilibrio in consequence of all these variations of the forces which solicit it. By the principle of virtual velocities the equations of this equilibrium may be obtained, which, combined with those arising from the connection of the parts of the system, give the differential equations of the motion of each of its points. In like manner the laws of the motion of fluids may be reduced to those of their equilibrium. In this case the conditions relative to the connection of the parts of the system are reduced to this; that if the fluid be incompressible, the

volume of any particle of it must remain constantly the same, and that it will depend on the pression according to some given law if the fluid be compressible and elastic. The equations which express these conditions, and the variations in the motion of the fluid, contain the partial differences of the co-ordinates of the particle, taken either with relation to the times or to the primitive co-ordinates. The integration of this species of equations is attended with great difficulties, and can only be effected in certain particular cases, relating to the motion of ponderable fluids in vases, to the theory of sounds, and to the oscillations of the atmosphere and ocean.

The consideration of the differential equations of the motion of bodies, has led to the discovery of several very general and useful principles in mechanics, and which are an extension of those which have been developed already relatively to the motion or point in the second chapter

of the second Book. A material point moves uniformly in a straight line, if not induced to quit it by the action of some foreign cause. In a system of bodies which act on each other, without being subject to the action of foreign causes, the common centre of gravity moves uniformly in a straight line, and its motion is the same as if all the bodies were supposed united in this point, and the forces which solicit them immediately applied to it, so that the quantity and direction of the resulting force remains always the same.

We have seen that the radius vector of a body, solicited by a force directed towards a fixed point, describes areas proportional to the times. If we suppose a system of bodies acting one upon the other in any manner whatever, and solicited by a force directed towards a fixed point, if from this point a radius vector be drawn to each of them, and these radii projected on an invariable plane passing through the point, the sum of the products of the

mass of each body, by the area traced by the projection of its radius vector, is proportional to the time. In this consists the principle of the *conservation of areas.*

The *vis viva* of a system of bodies is the sum of the products of the mass of each body by the square of its velocity. When a body moves on a line or surface without being subject to a foreign action, this *vis viva* is constantly the same, since the velocity is constant. If the bodies of a system experience no other actions than those arising from these mutual tractions and pressions, either directly or by the intermediate agency of rods or threads which are inextensible and unelastic, the *vis viva* of the system remains constant, even in the case where several of the bodies are constrained to move in curved lines or surfaces. This is the principle of the conservation of the *vis viva.*

In the motion of a point solicited by forces of any kind, the variation of the *vis viva* is equal to twice the sum of the

products of the mass of a point by each of the accelerating forces, multiplied respectively by the elementary quantities which the point advances towards their origins. In the motion of any system, the double of the sum of all these products is the variation of the *vis viva* of the system.

When the *vis viva* attains its maximum or minimum, this variation is nothing, the system then, according to the principle of virtual velocity, will be in equilibrio, in consequence of the accelerating forces which solicit it; and thus in all the situations which a system takes successively, that where the *vis viva* is the greatest or least, is that in which it will remain in equilibrio. There is, besides, this remarkable circumstance, that if in this situation the *vis viva* is constantly a maximum, whatever be the velocities of the bodies when they arrive at it, the equilibrium will be stable; but it will not be so if the *vis viva* be constantly a minimum.

This is an evident consequence of what has been stated above, relative to the simple oscillation of a system of bodies.

Finally, we have seen in the second Chapter, that the sum of the integrals of the products of the mass of each body of the system, by its velocity, and by the element of the curve which it describes, is a minimum; this constitutes the principle of the least action, which principle differs from that of the uniform motion of the centre of gravity, of the conservation of areas, and of the *vis viva*, in this, that these principles are the real integrals of the differential equations of the motion of bodies, whereas that of the least action is only a singular combination of these equations.

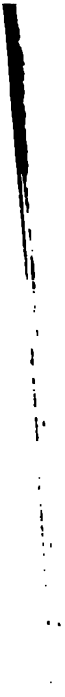
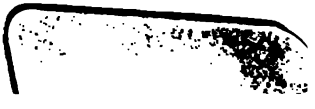
Another important remark remains to be made upon the extent of these different principles—that of the uniform motion of the centre of gravity of a system of bodies, and of the conservation of areas, subsist even in the case, when by the

mutual actions of the bodies there happens a sudden change in their motion, which renders these principles extremely useful in many circumstances; but the principle of the conservation of the *vis viva*, and that of the least action, require that the variations in the motion of the system take place by imperceptible degrees.

When the system suffers a sudden change, either by the mutual action of the bodies, or by meeting with obstacles, the *vis viva* receives at each of these changes a diminution equal to the sum of the products of each mass, multiplied by the sum of the squares of the variations which this change effects in its velocity, decomposed parallel to any three axes perpendicular with each other.

All these principles would subsist, regard being had to the relative motion of the bodies of a system, if it was carried on by a general motion common to the foci of the forces which we have supposed fixed. They will likewise take place in the rela-

tive motion of bodies upon the surface of the earth, for it is impossible to judge of the relative motion of a system of bodies by the appearance only of its relative motion.

END OF VOL. I.

W. Flint, Printer, Old Bailey.